Network Science

Depth-First Search

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Summary

1. Depth-First Search (DFS) Algorithm
2. Example
3. Applications

Depth-First Search (DFS) Algorithm

Depth-First Search

Needs:

- adjacency lists

Provides:

- edge classification
- cycle detection
- topological sort

Algorithm — Core

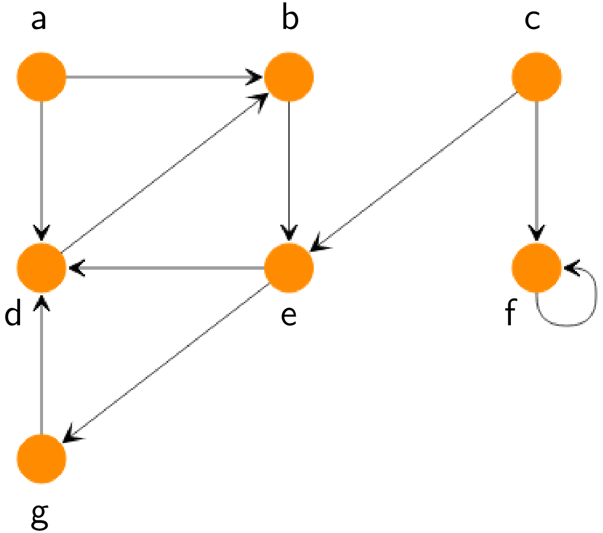
```
function DFS-Visit(u, Adj)
    for v in Adj[u] do
        if v not in parent then
            parent[v] $\leftarrow$ u
            DFS-Visit(v, Adj)
        end if
    end for
end function
```

Algorithm — Main

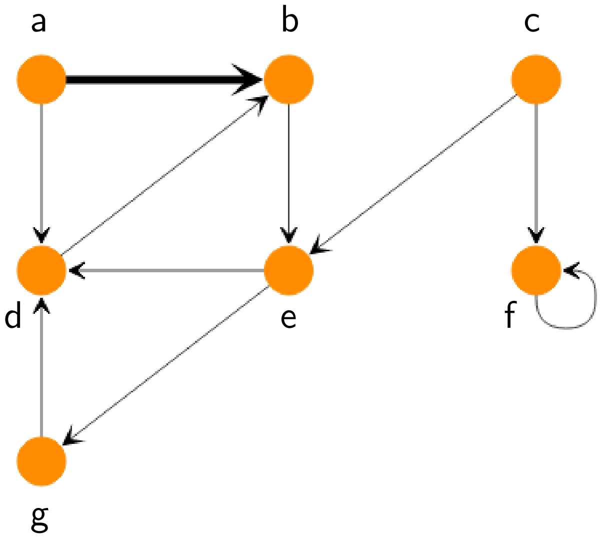
```
function DFS(V, Adj)
    parent ← {}
    for v in V do
        if v not in parent then
            parent[v] ← None
            DFS-Visit(v, Adj)
        end if
    end for
end function
```

Example

Example: directed graph



Example: directed graph



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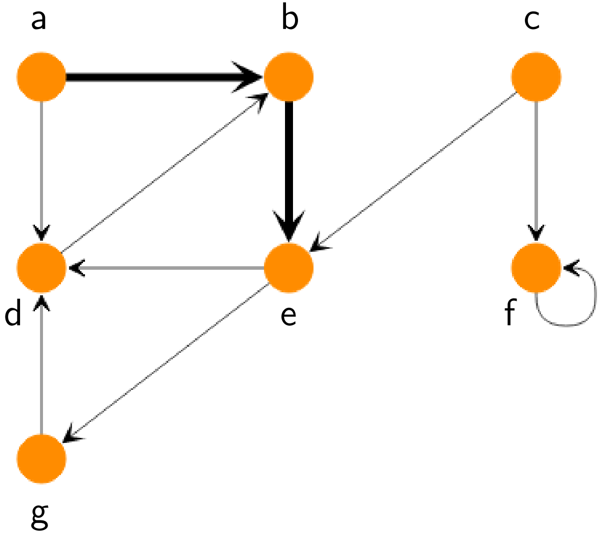

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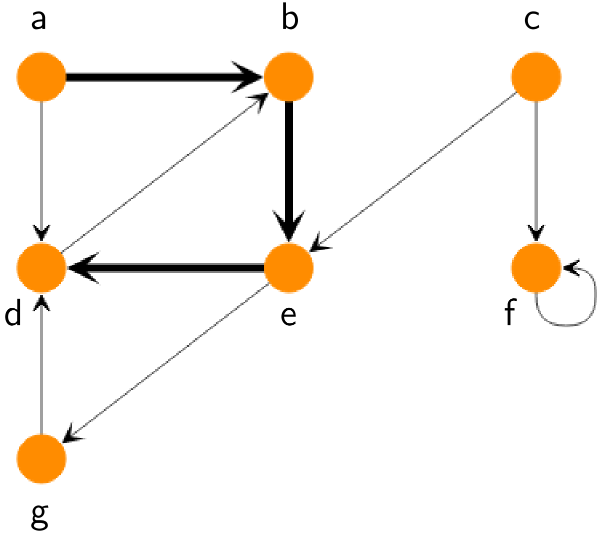

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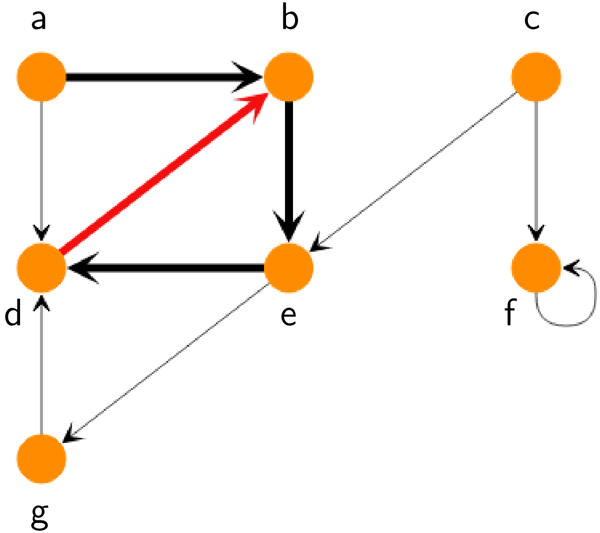

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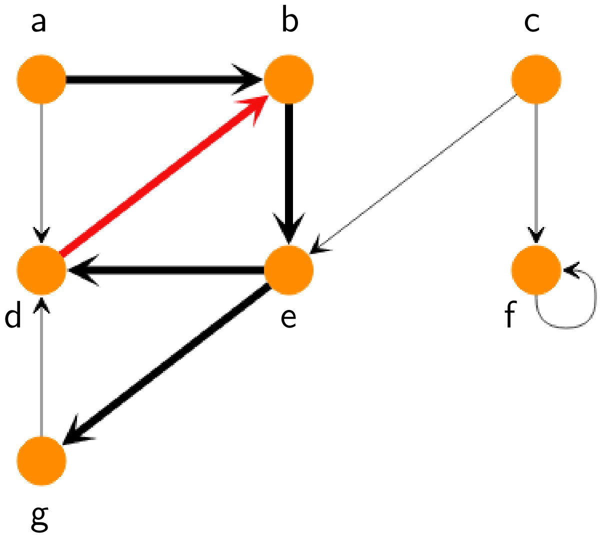

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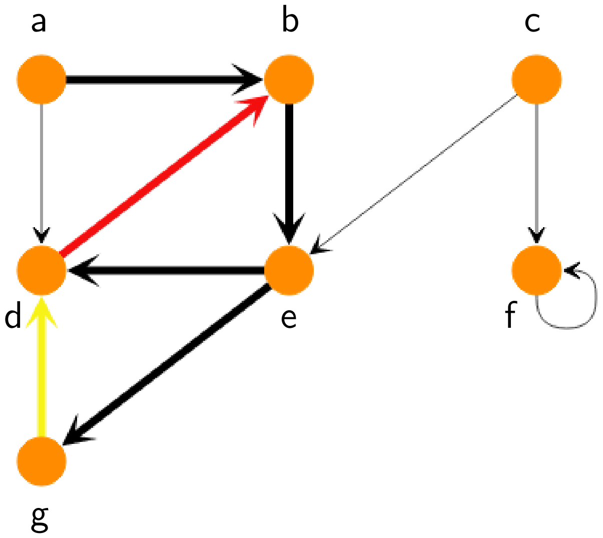

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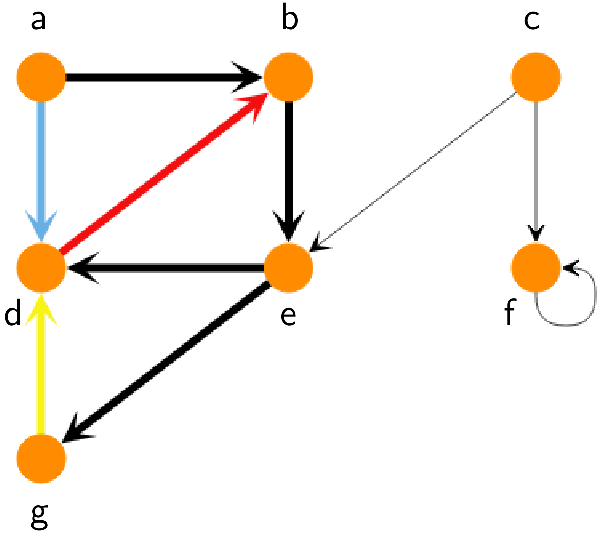

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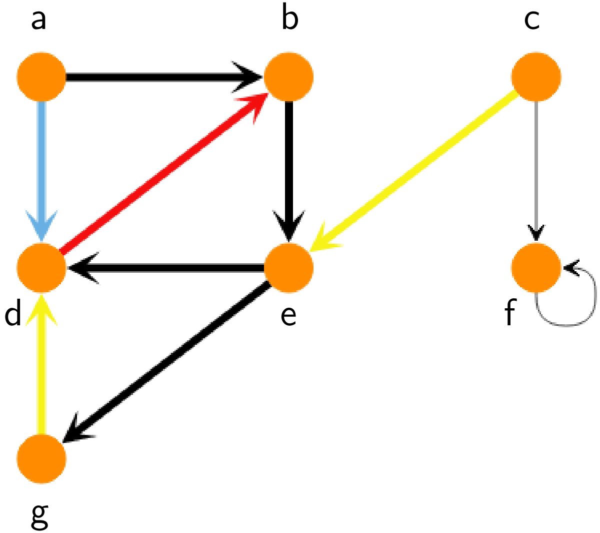
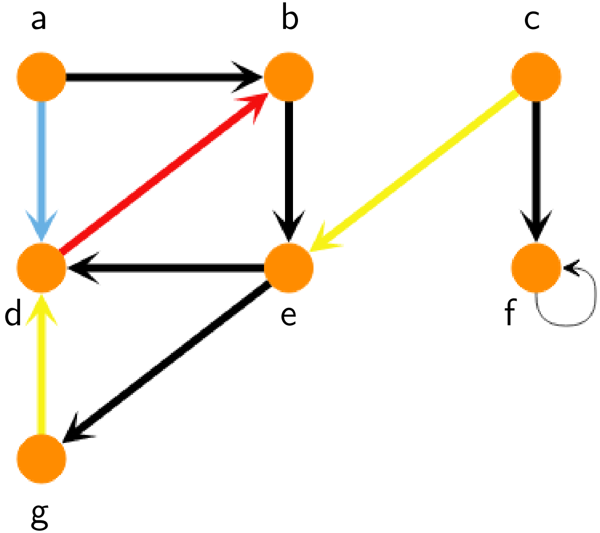
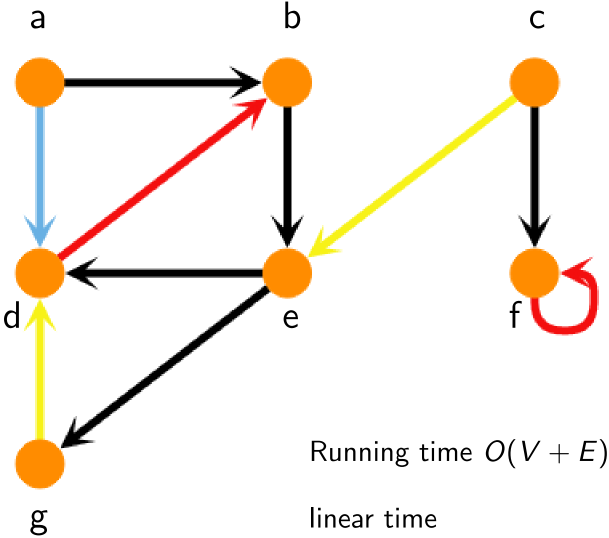


Example: directed graph



Example: directed graph



Running time $O(V + E)$

linear time

Applications

Edge classification

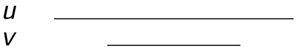
Depends on DFS itself, not just graph

Types of edges:

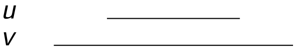
- tree edges
- forward edges
- backward edges
- cross edges

Algorithm additions

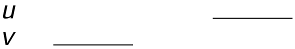
- Starting and ending times
  - useful to classify edges
- forward edges: $u \to v$ with



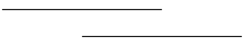
- backward edges: $u \to v$ with



- cross edges: $u \to v$ with



- impossible:



Undirected graph

- no forward edges
- no cross edges

Cycle detection

Graph has a cycle $\Longleftrightarrow$ DFS has a backward edge

Topological sorting

Premises:

- Acyclic graphs
- Tasks that depend on one another

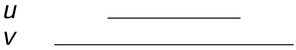
Results:

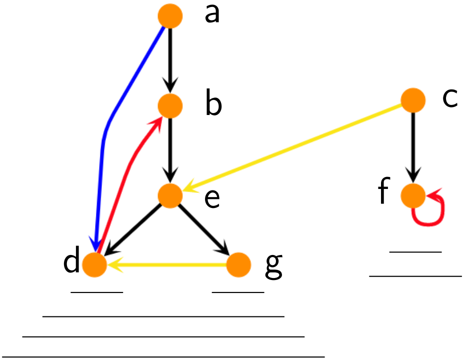
- Topological sort: Safe order for the tasks
- DFS: reverse order of finishing times

Cycle detection

DFS has a backward edge $\Longrightarrow$ Graph has a cycle

backward edge: $u \to v$ with





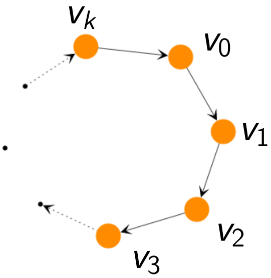
u starts while v active

$\Longrightarrow$

there is a path from v to u

Cycle detection

Graph has a cycle $\Longrightarrow$ DFS has a backward edge

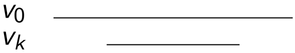


v_0: first visited vertex in cycle

$v_1, v_2, v_3, \ldots, v_k$: start after v_0

$v_1, v_2, v_3, \ldots, v_k$: start before v_0 finishes

Therefore,



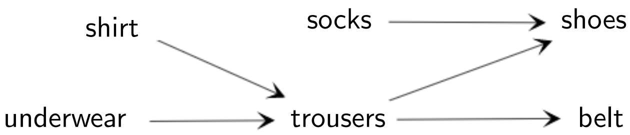
and $v_k \to v_0$ is a backward edge

Topological sorting

Example: getting dressed

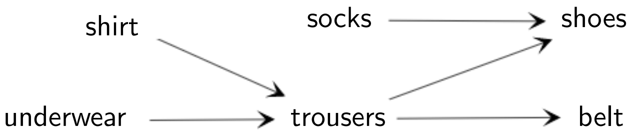
socks
shoes
trousers
belt
shirt
underwear

socks → shoes
underwear → trousers
shirt → trousers
trousers → belt
trousers → shoes



Topological sorting

Example: getting dressed



shoes, socks, belt, trousers, underwear, shirt

←

belt, shoes, trousers, shirt, socks, underwear

←