Design of Pattern Classifiers using Optimum-Path Forest with Applications in Image Analysis

Alexandre Xavier Falção

Visual Informatics Laboratory - Institute of Computing - University of Campinas

afalcao@ic.unicamp.br

www.ic.unicamp.br/~afalcao/talks.html

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- The applications are in many fields of the sciences and engineering.
- Our main focus has been on image analysis, where samples may be pixels, images, or image objects (e.g., contours, regions).

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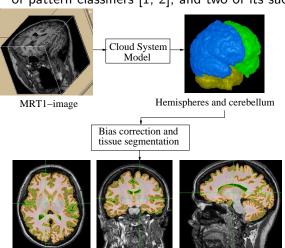
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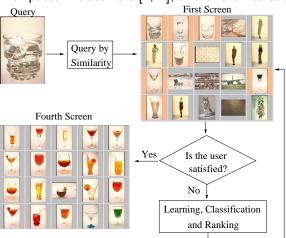
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- Brain tissue segmentation in 3D MRimages [3, 4].
- Content-based image retrieval using relevance feedback [5].

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- The method for brain tissue segmentation is fully automatic, accurate, and one of the fastest approaches to date.
- The method for CBIR can satisfy the user in a few iterations of relevance feedback with significant gain in effectiveness over other state-of-the-art approaches.

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- Conclusions and open problems.

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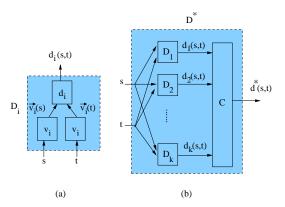
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- Simple descriptors may require more complex distance functions to compare color properties [15], shape characteristics [16], and texture information [17] between samples.

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We have found C by genetic programming [18].



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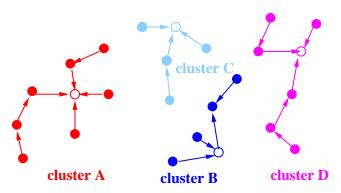
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- The result is an optimum-path forest (a pattern classifier) with roots in S.
- The class/cluster label of a new sample $t \in \mathbb{Z} \setminus \mathcal{T}$ is obtained from $d^*(s,t)$ using some samples $s \in \mathcal{T}$ and local attributes of the forest [4, 13].



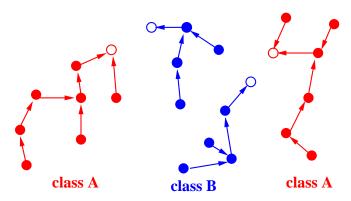
In unsupervised learning, each prototype propagates a distinct cluster label to the remaining training nodes of its tree.



Each cluster is one optimum-path tree.



In supervised learning, each prototype propagates its class label to the remaining training nodes of its tree.



Each class is one optimum-path forest.



• Training essentially maximizes (minimizes) a connectivity map

$$V(t) = \max_{\forall \pi_t \in \Pi(\mathcal{T}, \mathcal{A}, t)} \{ f(\pi_t) \}$$

by considering the set $\Pi(\mathcal{T}, \mathcal{A}, t)$ of all paths with terminus $t \in \mathcal{T}$ and path-value function f.

Methodology

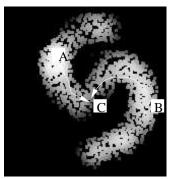
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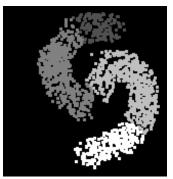
by considering the set $\Pi(\mathcal{T}, \mathcal{A}, t)$ of all paths with terminus $t \in \mathcal{T}$ and path-value function f.

 The solution is obtained by dynamic programming (i.e., Dijkstra's algorithm extended to multiple sources and more general path-value functions) [19].

For clustering, we can estimate a probability density function (pdf) and the maxima of the pdf compete with each other, such that each cluster will be an optimum-path tree rooted at one maximum of the pdf.

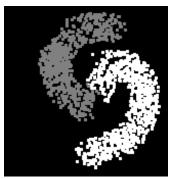


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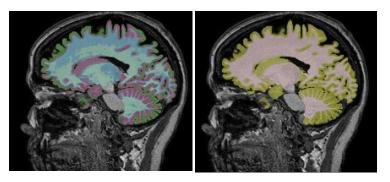
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Prior information is usually required to assign class labels to the clusters.



The unlabeled training samples form a knn-graph $(\mathcal{T}, \mathcal{A}_k)$ with adjacency relation

 A_k : $(s,t) \in A_k$ (or $t \in A_k(s)$) if t is k nearest neighbor of s using the underlying distance space.

The best value of k is the one whose clustering produces a minimum normalized graph cut in $(\mathcal{T}, \mathcal{A}_k)$.

The graph is weighted on the arcs $(s,t) \in A_k$ by $d^*(s,t)$ and on the nodes by the pdf $\rho(s)$.

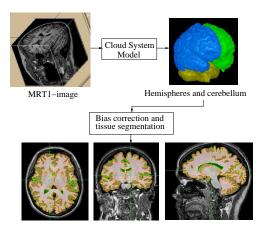
$$\rho(s) = \frac{1}{\sqrt{2\pi\sigma^2}|\mathcal{A}_k(s)|} \sum_{\forall t \in \mathcal{A}_k(s)} \exp\left(\frac{-d^{*2}(s,t)}{2\sigma^2}\right)$$

where $\sigma = \frac{d_f}{3}$ and $d_f = \max_{\forall (s,t) \in \mathcal{A}_k} \{d^*(s,t)\}$. The pdf is usually normalized within an interval [1,K] to facilitate the choice of relevant maxima and extra arcs are added in \mathcal{A}_k to guarantee arc symmetry on the plateaus of the pdf.

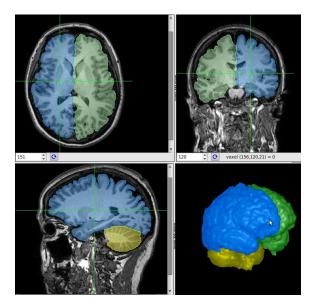
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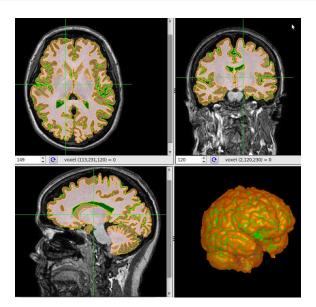
$$f_{\mathsf{min}}(\langle t \rangle) = \left\{ egin{array}{ll}
ho(t) & ext{if } t \in \mathcal{S} \\
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ight.$$
 $f_{\mathsf{min}}(\pi_s \cdot \langle s, t \rangle) = \min\{f_{\mathsf{min}}(\pi_s),
ho(t)\}$

where $\mathcal S$ is the root set (maxima) found on-the-fly.



After brain segmentation and bias correction, brain voxels are first classified into CSF or GM+WM and then classified into GM or WM, because the method requires different parameters in each case.





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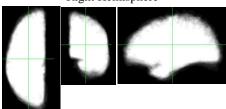
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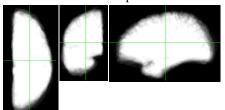
The method captures possible shape variations and groups them into a few CSMs.

Orthogonal cuts of the 3D clouds

Right Hemisphere



Left Hemisphere



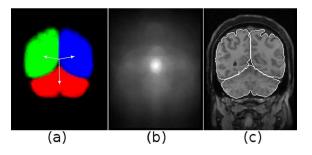
Cerebellum



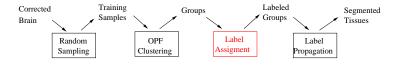


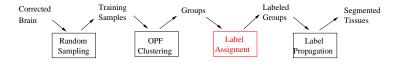


Brain segmentation consists of a search for the translation to the image location which produces the highest score, when the reference point of the most suitable CSM is at that position.



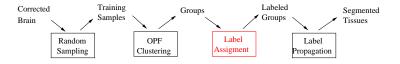
We use four 3D CSMs for brain segmentation.



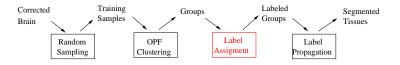


In each step of voxel clustering.

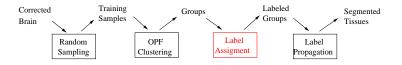
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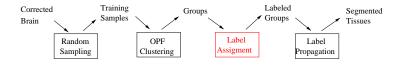


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- A small training set $\mathcal{T} \subset \mathcal{Z}$ is obtained by random sampling.

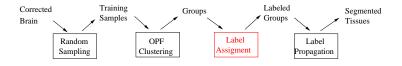


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- ullet A small training set $\mathcal{T}\subset\mathcal{Z}$ is obtained by random sampling.
- The OPF clustering can find groups of voxels in T, mostly from a same class.

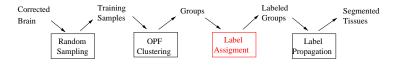




• Class labels are assigned to each group and propagated to the remaining voxels in \mathcal{Z} .

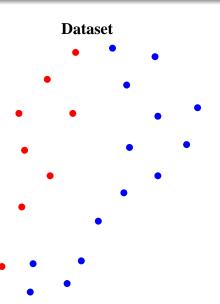


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- For MRT1-images, label assignment is done from the darkest to the brightest cluster until the size proportion p between the classes is the closest to a previously estimated value p_T , which is obtained by automatic thresholding.

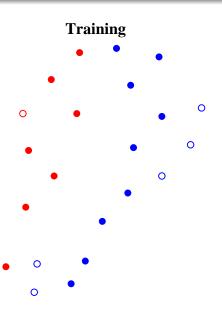


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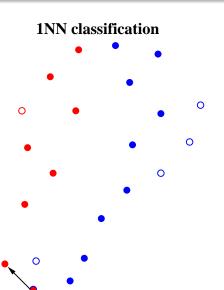
The entire process takes less than 2 minutes in a Core i7 PC, being about 1 minute for brain segmentation using 4 CSMs, and less than 1 minute for bias correction and tissue segmentation.



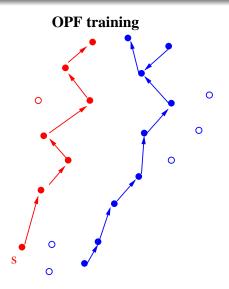
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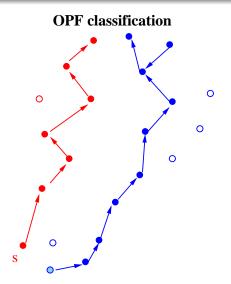
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- Classification by nearest neighbor fails, when training samples are close to test samples (empty bullets) from other classes.



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- V(s) can then be used to reduce the power of s to classify new samples.

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- For a given set $\mathcal{S} \subset \mathcal{T}$ of prototypes from all classes, the connectivity map V(t) is minimized for

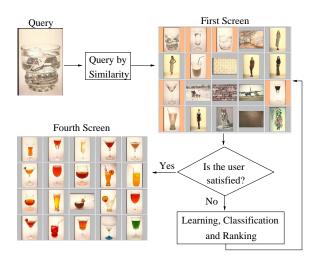
$$f_{\sf max}(\langle t \rangle) = \begin{cases} 0 & \text{if } t \in \mathcal{S} \\ +\infty & \text{otherwise} \end{cases}$$

 $f_{\sf max}(\pi_s \cdot \langle s, t \rangle) = \max\{f_{\sf max}(\pi_s), d^*(s, t)\}$

where $d^*(s, t)$ is the distance between s and t.



Content-based image retrieval



The relevant and irrelevant images are the nodes of $(\mathcal{T}, \mathcal{A})$, whose arcs are weighted by a color descriptor [15].

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In each iteration of relevance feedback.

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- The relevant candidates are ordered based on their average distances to the relevant prototypes.

Show software.

Dealing with large training sets

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- We have fixed the size of \mathcal{T}^* and replaced classification errors in $\mathcal{T} \setminus \mathcal{T}^*$ by non-representative samples in \mathcal{T}^* [1].
- We have recently devised a method that learns a minimum size of \mathcal{T}^* in order to obtain a desired classification accuracy on $\mathcal{T} \backslash \mathcal{T}^*$.

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- The C source code is available in www.ic.unicamp.br/~afalcao/libopf.



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Thank you

- J.P. Papa, A.X. Falcão, and C.T.N. Suzuki.
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