Volumetric Image Visualization

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From 3D to 2D

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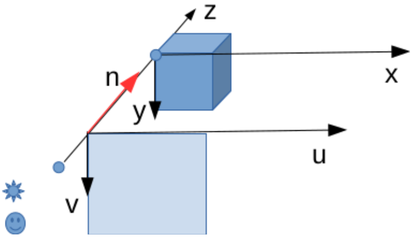
- Rendering — a process that creates an image (rendition) in a viewing plane with some object information from a 3D image (scene).
- Rendering with orthogonal projection — a geometric transformation that avoids distortions in the interpretation of the scene.
- A simple rendering example — how to draw the wireframe of a scene.

Rendering

By using some illumination model, one can simulate the transformation of a scene followed by the projection onto a viewing plane *uv* of the light reflected on the surface of each object inside the scene.



An observer and a white light source may be assumed to be at an infinite distance ($z = -\infty$) to the viewing plane. One can also simulate the inverse transformation of the observer, light source, and viewing plane.

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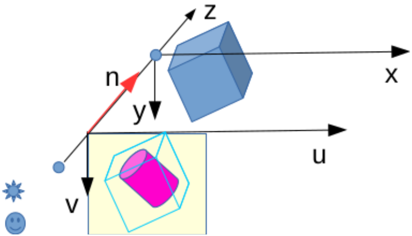

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Orthogonal projection

- A projection is a change from the scene coordinate system (x, y, z) to the coordinate system (u, v, n) of the viewing plane, where $\mathrm{n} = (0, 0, 1, 0)$ is the viewing direction and $-\mathrm{v} = (0, -1, 0, 0)$ is the view-up vector.

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- By rotating a scene, with diagonal d, around its center $c = (x_c, y_c, z_c, 1)$ and axes x (*tilt*) and y (*spin*), one obtains its orthogonal projection in uv as

$$
\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \mathrm{T}(\frac{d}{2}, \frac{d}{2}, \frac{d}{2})\mathrm{R}_y(\beta)\mathrm{R}_x(\alpha)\mathrm{T}(-x_c, -y_c, -z_c) \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}
$$

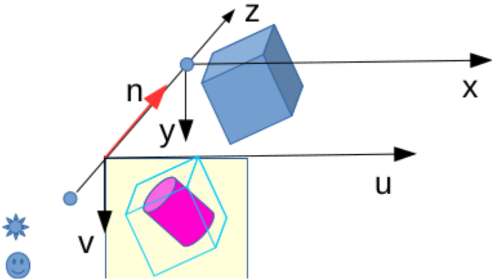
$$
u = x_2
$$

$$
v = y_2
$$

where changes in $\mathrm{T}(\frac{d}{2}, \frac{d}{2}, \frac{d}{2})$ and on the position of uv might cause clipping and cut of the scene.

Orthogonal projection

The value z_2 represents the distance between xy and the object's surface, so those distance values may be stored in a z-buffer and used for hidden surface removal or, together with the illumination model, for showing the closer objects brighter in the rendition $\hat{J} = (D_J, J)$, with $d \times d$ pixels to avoid clipping.



The bounding box of the scene (cyan) is called its wireframe.

Drawing the wireframe of the scene

- Consider the problem of drawing the visible edges of the scene's wireframe in uv, for different tilt and spin angles, by assuming the faces of the scene are opaque.

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- Consider the problem of drawing the visible edges of the scene's wireframe in uv, for different tilt and spin angles, by assuming the faces of the scene are opaque.

- As strategy, one can

  - first determine which faces are visible after transformation and,

  - for each edge, if both of its extreme points belong to a visible face,

    - they must be projected and

    - a line must be drawn between them in the uv plane.

Graphical context

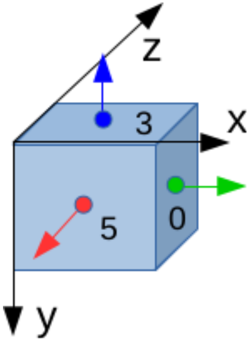
A first step in rendering is to define the graphical context, which must store all information needed for the rendering.

- The transformations that will be applied to points and vectors, respectively.

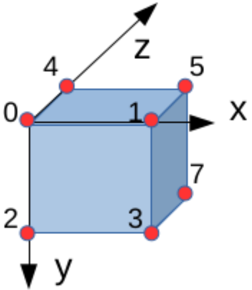
$$
\begin{aligned}
\Phi &= \mathrm{T}(\frac{d}{2},\frac{d}{2},\frac{d}{2})\mathrm{R}_y(\beta)\mathrm{R}_x(\alpha)\mathrm{T}(-x_c,-y_c,-z_c) \\
\Phi_r &= \mathrm{R}_y(\beta)\mathrm{R}_x(\alpha)
\end{aligned}
$$

Graphical context

- A set $\mathcal{F}$ with the faces of the scene, as represented by their centers and normal vectors, respectively:
  0: $(n_x - 1, \frac{n_y}{2}, \frac{n_z}{2}, 1)$ and $(1, 0, 0, 0)$; 1: $(0, \frac{n_y}{2}, \frac{n_z}{2}, 1)$ and $(-1, 0, 0, 0)$; 2: $(\frac{n_x}{2}, n_y - 1, \frac{n_z}{2}, 1)$ and $(0, 1, 0, 0)$;
  3: $(\frac{n_x}{2}, 0, \frac{n_z}{2}, 1)$ and $(0, -1, 0, 0)$; 4: $(\frac{n_x}{2}, \frac{n_y}{2}, n_z - 1, 1)$ and $(0, 0, 1, 0)$; and 5: $(\frac{n_x}{2}, \frac{n_y}{2}, 0, 1)$ and $(0, 0, -1, 0)$.



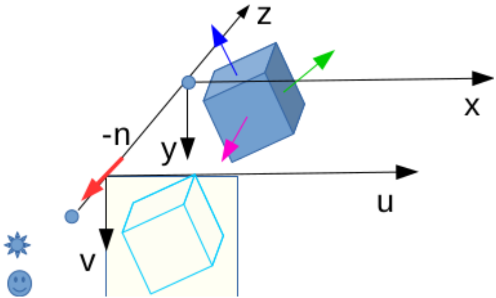
- A set $\mathcal{V}$ with the vertices of the scene:
  $0: (0, 0, 0, 1)$; $1: (n_x - 1, 0, 0, 1)$; $2: (0, n_y - 1, 0, 1)$;
  $3: (n_x - 1, n_y - 1, 0, 1)$; $4: (0, 0, n_z - 1, 1)$;
  $5: (n_x - 1, 0, n_z - 1, 1)$; $6: (0, n_y - 1, n_z - 1, 1)$;
  $7: (n_x - 1, n_y - 1, n_z - 1, 1)$;



- A set $\mathcal{E}$ with the edges of the scene by indicating which vertices compose each edge (e.g., edge 0 connects vertices 0 and 1).

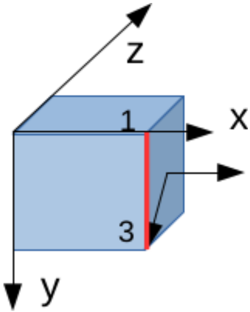
A visible face

A face $f \in \mathcal{F}$ is visible if, after transformation $\Phi_r(f.\text{n})$ of its normal vector $f.\text{n}$, the inner product $\langle \Phi_r(f.\text{n}), -\text{n} \rangle > 0$, where $-\text{n} = (0, 0, -1, 0)$.



A point in a face

The extreme points $e.p_1$ and $e.p_n$ of an edge $e \in \mathcal{E}$ fall in a face $f \in \mathcal{F}$ with center at $f.c$, iff $\langle e.p_1 - f.c, f.\mathrm{n} \rangle = 0$ and $\langle e.p_n - f.c, f.\mathrm{n} \rangle = 0$.



Drawing an edge of a visible face

- By identifying that a face $f \in \mathcal{F}$ is visible and an edge $e \in \mathcal{E}$ is part of that face, we must transform the points of e, by $p_1 \leftarrow \Phi(e.p_1)$ and $p_n \leftarrow \Phi(e.p_n)$, and draw a line in uv (i.e., image $\hat{J} = (D_J, J)$) from $p_1 = (u_{p_1}, v_{p_1})$ to $p_n = (u_{p_n}, v_{p_n})$.

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- This line can be drawn by using the *Digital Differential Analyzer* (DDA) algorithm.

The DDA algorithm in 2D

- Let $p_k = (u_{p_k}, v_{p_k})$, $k = 1, 2, \ldots, n$, be the n points drawn from p_1 to p_n in uv. Each subsequent point $p_{k+1} = (u_{p_{k+1}}, v_{p_{k+1}})$ is obtained by

$$(u_{p_{k+1}}, v_{p_{k+1}}) = (u_{p_k}, v_{p_k}) + (d_u, d_v),$$

where the displacement (d_u, d_v) follows the direction and orientation of the line segment from p_1 to p_n. These points must also be approximated to pixels in D_J.

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- In order to avoid that a pixel be visited multiple times, d_u and d_v can be computed as $d_a = sign(a_{p_n} - a_{p_1}) \in \{-1, 1\}$, $a \in \{u, v\}$.

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- The maximum intensity H can be used to draw the lines in the output image $\hat{J} = (D_J, J)$.

The DDA algorithm in 2D

Input: Points p_1 and p_n, and intensity H.
Output: Image $\hat{J} = (D_J, J)$.

```
If $p_1 = p_n$ then set $n \leftarrow 1$.
Else
    set $D_u \leftarrow u_{p_n} - u_{p_1}$ and $D_v \leftarrow v_{p_n} - v_{p_1}$.
    If $|D_u| \geq |D_v|$ then
        set $n \leftarrow |D_u| + 1$, $d_u \leftarrow sign(D_u)$, and $d_v \leftarrow \frac{d_u D_v}{D_u}$.
    Else
        set $n \leftarrow |D_v| + 1$, $d_v \leftarrow sign(D_v)$, and $d_u \leftarrow \frac{d_v D_u}{D_v}$.
Set $p = (u_p, v_p) \leftarrow (u_{p_1}, v_{p_1})$.
For each $k = 1$ to $n$, do
    Set $J(p) \leftarrow H$.
    Set $p = (u_p, v_p) \leftarrow (u_p, v_p) + (d_u, d_v)$.
```

Adding attributes to the rendering

- The lines may be drawn with thickness $2r$ by changing $J(p) \leftarrow H$ to $\forall q \in \mathcal{A}_r(p), J(q) \leftarrow H$, where $q \in \mathcal{A}_r(p)$ if $1 \leq \|q - p\| \leq r$.

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- One can also draw colored lines by painting their red, green, and blue values in three bands of a colored image $\hat{J} = (D_J, \text{J})$.

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- Let's implement this simple algorithm as exercise?

Algorithm to draw a wireframe

Input: Scene $\hat{I} = (D_I, I)$, angles α, and β.
Output: Rendition $\hat{J} = (D_J, J)$ of the wireframe.

1. create a graphical context with $\mathcal{F}$, $\mathcal{E}$, H, Φ, and Φ_r from D_I, α, and β.
2. for each $f \in \mathcal{F}$ do
3.   if $\langle \Phi_r(f.\mathrm{n}), (0, 0, -1, 0) \rangle > 0$ then
4.   for each edge $e \in \mathcal{E}$ do
5.   if $\langle e.p_1 - f.c, f.\mathrm{n} \rangle = 0$ and $\langle e.p_n - f.c, f.\mathrm{n} \rangle = 0$ then
6. $p_1 \leftarrow \Phi(e.p_1)$ and $p_n \leftarrow \Phi(e.p_n)$
7.   DrawLine2D($\hat{J}, p_1, p_n, H$)