# Volumetric Image Visualization 

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## Introduction

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This lecture covers object transparency in the surface rendering algorithm.


## Ray casting with transparency

Let $0 \leq o_{i} . \alpha \leq 1$ be the opacjty of object $o_{i}$ and $p_{i}^{\prime}$,
$i=1,2, \ldots, k$, be the first intersection points of a viewing ray and
$1 \leq k \leq c$ distinct objects of the scene.


- We will assume that refraction does not change the direction of the ray.


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- We will assume that refraction does not change the direction of the ray.
- Being the first intersection points for each object guarantees that they are visible surfaces - i.e., $0<\left\langle o_{i} . \mathbf{n}\left(p_{i}^{\prime}\right),-\mathbf{n}^{\prime}\right\rangle \leq 1$, $1 \leq i \leq k-$, depending on the transparencies of their occluding objects $o_{j}, 1 \leq j<i$, only.


## Ray casting with transparency

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- Due to color reflectance, $\mathbf{c}\left(p_{i}^{\prime}\right)=r\left(p_{i}^{\prime}\right) \times o_{i} . \mathbf{r}$, where $o_{i} \cdot \mathbf{r}=\left(o_{i} \cdot R, o_{i} \cdot G, o_{i} \cdot B\right)$, is the reflected light by object $o_{i}$.


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- Due to opacity, only a portion $o_{1} . \alpha \times \mathbf{c}\left(p_{1}^{\prime}\right)$ returns to the observer from $o_{1}$, a portion $\left(1-o_{1} . \alpha\right) \times o_{2} . \alpha \times \mathbf{c}\left(p_{2}^{\prime}\right)$ returns from $\mathrm{o}_{2}$, and so on, such that the light reflected towards the observer is:

$$
\mathbf{c}\left(p_{1}^{\prime}, p_{2}^{\prime}, \ldots, p_{k}^{\prime}\right)=\sum_{i=1}^{k}\left[\Pi_{j=1}^{i-1}\left(1-o_{j} . \alpha\right)\right] \times o_{i} . \alpha \times \mathbf{c}\left(p_{i}^{\prime}\right)
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- For the rendition $\hat{\jmath}=\left(D_{J}, \mathbf{J}\right), \mathbf{J}(p) \leftarrow \mathbf{c}\left(p_{1}^{\prime}, p_{2}^{\prime}, \ldots, p_{k}^{\prime}\right)$.


## Graphical context

The graphical context gc may then store.

- Transformations $\phi^{-1}$ and $\phi_{r}^{-1}$.
- Phong parameters $k_{a}, k_{s}, k_{d}, n_{s}, H, d_{\text {min }}, d_{\text {max }}$.
- The scene $\hat{I}=\left(D_{I}, I\right)$ and its object label image $\hat{L}\left(D_{I}, L\right)$, such that $L(p)=i, i=1,2, \ldots, c$, when $p \in D_{l}$ belongs to one of the $c$ objects, or $L(p)=0$ otherwise.


## Graphical context

- look-up table $\mathbf{n}[i]$ with pre-computed normal vectors.
- A normal index map $N$ with the address $N(p)=i$ of the normal vector in $\mathbf{n}[i]$ for each boundary voxel $p \in D_{l}$, or nil when $p$ is not a boundary voxel.
- A table with the visual attributes for each object $o_{i}$, $i=1,2, \ldots, c$,
- color $o_{i} \cdot \mathbf{r}=\left(o_{i} \cdot R, o_{i} \cdot G, o_{i} \cdot B\right)$, where $o_{i} \cdot X \in[0,1]$,
- opacity $0<o_{i} . \alpha \leq 1$, and
- visibility $o_{i} . v \in\{0,1\}$.


## Surface rendering algorithm with opacities

Input: Graphical context $g c$, and viewing angles $\alpha$ and $\beta$.
Output: Rendition $\hat{\jmath}=\left(D_{J}, \mathbf{J}\right)$.
$1 \mathbf{n}^{\prime} \leftarrow \phi_{r}^{-1}(\mathbf{n})$, where $\mathbf{n}=(0,0,1,0)$.
2 For each $p \in D_{J}$ do
$3 \quad p_{0} \leftarrow \phi^{-1}(p)$.
$4 \quad$ Find $\mathcal{P}=\left\{p_{1}, p_{n}\right\}$ by solving $\left\langle p_{0}+\lambda \mathbf{n}^{\prime}-f . c, f . \mathbf{n}\right\rangle=0$ for each face $f \in \mathcal{F}$ of the scene, whenever they exist.
5 if $\mathcal{P} \neq \emptyset$ then
6
$\mathbf{J}(p) \leftarrow$ ComputeColorAlongRay $(g c, \mathcal{P})$.

## Surface rendering algorithm with opacities

- ComputeColorAlongRay $(g c, \mathcal{P})$ is the DDA algorithm modified to compute the Phong's model with opacities along a viewing ray.

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- The accumulated transparency $\left[\Pi_{j=1}^{i-1}\left(1-o_{j} . \alpha\right)\right]$ can be stored in a variable $t$, initially set to 1 and used for early ray termination.


## Surface rendering algorithm with opacities

- A surface point flag $f_{s} \in\{0,1\}$ to stop the adjacency search that avoids missing a boundary voxel along the ray.



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- Object flags $o_{i} . f \in\{0,1\}, i=1,2, \ldots, c$, to avoid projecting more than one boundary voxel of a same object.

- The total color $\mathbf{c}_{t}=\mathbf{c}\left(p_{1}^{\prime}, p_{2}^{\prime}, \ldots, p_{k}^{\prime}\right)$ along the ray.


## Computing color along a viewing ray

1 If $p_{1}=p_{n}$ then set $n \leftarrow 1$.
2 Else
3 Set $D_{x} \leftarrow x_{p_{n}}-x_{p_{1}}, D_{y} \leftarrow y_{p_{n}}-y_{p_{1}}, D_{z} \leftarrow z_{p_{n}}-z_{p_{1}}$.
4 If $\left|D_{x}\right| \geq\left|D_{y}\right|$ and $\left|D_{x}\right| \geq\left|D_{z}\right|$ then
$5 \quad$ Set $n \leftarrow\left|D_{x}\right|+1, d_{x} \leftarrow \operatorname{sign}\left(D_{x}\right), d_{y} \leftarrow \frac{d_{x} D_{y}}{D_{x}}$, and $d_{z} \leftarrow \frac{d_{x} D_{z}}{D_{x}}$.
6 Else
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$$
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$$

## Computing color along a viewing ray

11 Set $k \leftarrow 1, t \leftarrow 1.0$, o o $. f \leftarrow 0, i=1,2, \ldots, c$,

$$
p^{\prime} \leftarrow\left(x_{p_{1}}, y_{p_{1}}, z_{p_{1}}\right), \text { and } \mathbf{c}_{t} \leftarrow(0,0,0)
$$

12 While $k \leq n$ and $t>\epsilon$, do
13 Set $p^{\prime} \leftarrow\left(\left\lceil x_{p^{\prime}}\right\rceil,\left\lceil y_{p^{\prime}}\right\rceil,\left\lceil z_{p^{\prime}}\right\rceil\right)$ and $f_{s} \leftarrow 0$.
14 For each $q \in \mathcal{A}_{1}\left(p^{\prime}\right)$ and while $f_{s}=0$, do
15 If $N(q) \neq$ nil and $o_{L(q)} \cdot f=0$ then
$16 \quad$ Set $f_{s} \leftarrow 1$.
17 If $o_{L(q)} \cdot v=1$ and $o_{L(q)} \cdot \alpha>0$, do

$$
\mathbf{c}_{t} \leftarrow \mathbf{c}_{t}+t \times o_{L(q)} \cdot \alpha \times r(q) \times o_{L(q)} \cdot \mathbf{r} .
$$

19

$$
t \leftarrow t \times\left(1-o_{L(q)} \cdot \alpha\right) \text { and } o_{L(q)} \cdot f \leftarrow 1
$$

20 Set $p^{\prime} \leftarrow\left(x_{p^{\prime}}, y_{p^{\prime}}, z_{p^{\prime}}\right)+\left(d_{x}, d_{y}, d_{z}\right)$
21 return $\mathbf{c}_{t}$.

## Result

Note that the rendition might change the original color of the objects.


