

# Volumetric Image Visualization

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In this lecture, we will learn how to

- quickly **estimate** a normal vector during the surface rendering algorithm, and
- **encode** a normal vector  $o \cdot \mathbf{n}(p')$ ,  $p' \in \mathcal{S}$ , by using only two bytes per boundary voxel.

# Introduction

In this lecture, we will learn how to

- quickly **estimate** a normal vector during the surface rendering algorithm, and
- **encode** a normal vector  $o \cdot \mathbf{n}(p')$ ,  $p' \in \mathcal{S}$ , by using only two bytes per boundary voxel.
- The lecture will also cover the creation of **stereo** renditions by surface rendering.

# Distance and index buffers

Instead of directly obtaining the Phong's shading  $r(p')$  as a result of ray casting from each pixel  $p \in D_J$ , one can quickly generate **distance**  $\hat{D} = (D_J, D)$  and **index**  $\hat{V} = (D_J, V)$  buffers from ray casting and use them to compute  $\hat{J} = (D_J, \mathbf{J})$ ,  $\mathbf{J}(p) \leftarrow r(p')(o.r_1, o.r_2, o.r_3)$ , afterwards.

$$r(p') = k_a r_a + r_d(p') (k_d \cos(\theta) + k_s \cos^{n_s}(2\theta))$$

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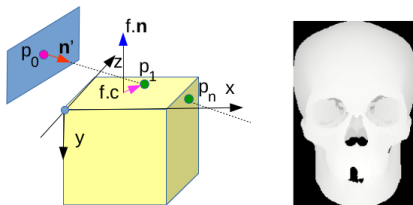
The distance map  $D$  can be used to estimate the **depth shading**  $r_d(p')$  while the index map  $V$  can be used to estimate the **normal vector** at  $p'$  (i.e., to obtain  $\theta$ ).

# Depth shading

The depth map stores the distance  $D(p) = \|p' - p_0\|$ ,  
 $p_0 = \phi^{-1}(p)$ , between the viewing plane and the surface point  $p'$   
that is reached from  $p \in D_J$ . The depth shading can be defined as

$$r_d(p') = H \frac{d_{\max} - D(p)}{d_{\max} - d_{\min}},$$

where  $d_{\min}$  and  $d_{\max}$  are the minimum and maximum values in  $D$ .



# Normal estimation from the index buffer

The index map stores the closest voxel

$V(p) = p' = (\lceil x_{p'} \rceil, \lceil y_{p'} \rceil, \lceil z_{p'} \rceil) \in \mathcal{S} \subset D_I$  reached from  $p \in D_J$ .

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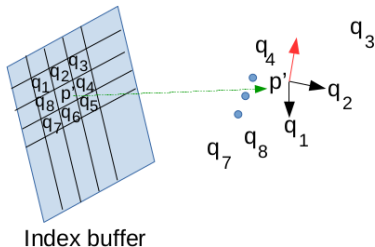
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- This allows normal estimation in the case of **opaque objects** only.
- One has to be careful to avoid computations with voxels from distinct objects and distant parts of the same object.
- This technique might create artifacts, but it performs a good job in most cases.
- It also allows direct access to  $p'$  when the user clicks on a pixel  $p$  in the rendition  $\hat{J} = (D_J, \mathbf{J})$ .

# Normal estimation from the index buffer

Let  $\mathcal{A}_r(p') = \{q \in \mathcal{S} \mid \|q - p'\| \leq r, L(q) = L(p')\}$  be an **ordered set** obtained from the 8-neighbors of  $p \in D_J$  in  $\hat{V}$  as visited in a counter-clockwise order. The normal vector  $\mathbf{n}(p')$  can be obtained from the subsequent spels  $q, q' \in \mathcal{A}_r(p') \mid q \prec q'$ , as

$$\mathbf{n}(p') = \frac{1}{|\mathcal{A}_r(p')|} \sum_{\forall q, q' \in \mathcal{A}_r(p') \mid q \prec q'} \frac{q - p'}{\|q - p'\|} \times \frac{q' - p'}{\|q' - p'\|}$$



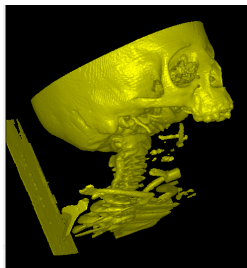
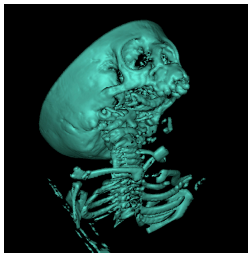
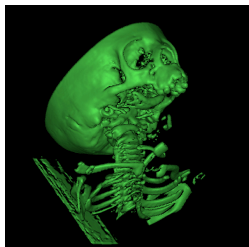
The distance  $r > 0$  should be a small value.

# Revisiting the surface rendering algorithm

- 01  $\mathbf{n}' \leftarrow \phi_r^{-1}(\mathbf{n})$ , where  $\mathbf{n} = (0, 0, 1, 0)$ .
- 02 For each  $p \in D_J$  do
- 03      $p_0 \leftarrow \phi^{-1}(p)$ .
- 04     Find  $\mathcal{P} = \{p_1, p_n\}$  by solving  $\langle p_0 + \lambda \mathbf{n}' - f.c, f.\mathbf{n} \rangle = 0$   
for each face  $f \in \mathcal{F}$  of the scene, whenever they exist.
- 05     if  $\mathcal{P} \neq \emptyset$  then
- 06          $p' \leftarrow \text{FindSurfacePoint}(\hat{L}, \mathcal{P})$ .
- 07         if  $p' \neq \text{nil}$  then  $D(p) \leftarrow \|p' - p_0\|$  and  $V(p) \leftarrow p'$ .
- 08 Compute  $d_{\min} = \min_{p \in D_J} \{D(p) > 0\}$  and  
 $d_{\max} = \max_{p \in D_J} \{D(p)\}$ .
- 09 For each  $p \in D_J$  do
- 10     Find  $o.\mathbf{n}(p')$ , where  $V(p) = p'$ , and  $r_d(p')$  from  $D(p)$ .
- 11      $\mathbf{J}(p) \leftarrow \mathbf{c}(p')$  by Phong's model.

# Comparison among the three approaches

Note that, for a same set  $\mathcal{S}$ , the index-based normal estimation (right) shows less holes than the scene-based (left) and object-based (center) approaches, because it might be using parts of the internal boundary near the external one.



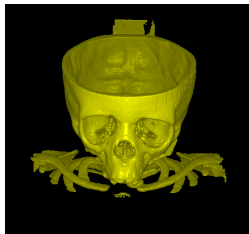
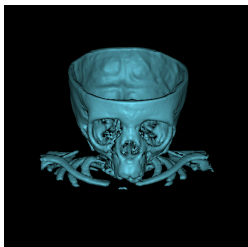
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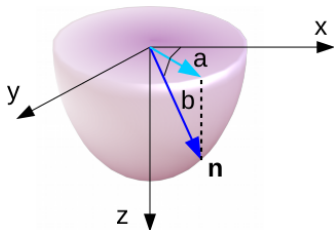
# Encoding the normal vectors

Let a **unit vector**  $\mathbf{n} = (n_x, n_y, n_z)$  be written in polar coordinates

$$n_x = \cos(b) \cos(a)$$

$$n_y = \cos(b) \sin(a)$$

$$n_z = \sin(b)$$



One may create a **look-up table** with unit vectors by varying  $a = 0, 1, \dots, 359$  and  $b = -90, -89, \dots, 90$ , as follows.

# Encoding the normal vectors

Let  $\mathbf{n}[i] = (n_x[i], n_y[i], n_z[i])$  be a look-up table of possible unit normal vectors.

01.  $i \leftarrow 1$
02. For  $b \leftarrow -90$  to  $90$  do
03.    $\gamma \leftarrow b \frac{\pi}{180}$ .
04.   For  $a \leftarrow 0$  to  $a = 359$  do
05.      $\alpha \leftarrow a \frac{\pi}{180}$ .
06.      $n_x[i] \leftarrow \cos(\gamma) \cos(\alpha)$
07.      $n_y[i] \leftarrow \cos(\gamma) \sin(\alpha)$
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The idea is to store one index in 2 bytes per boundary voxel  $p' \in \mathcal{S}$ .

# Encoding the normal vectors

Therefore, for each voxel  $p' \in \mathcal{S}$  of each object  $o$ ,

- one can estimate its unit normal vector  $o.\mathbf{n}(p')$ ,
- find the index  $i^*$  of its closest unit normal  $\mathbf{n}$  in the look-up table,

$$i^* = \arg \max_i \{ \langle o.\mathbf{n}(p'), \mathbf{n}[i] \rangle \},$$

- and store the index  $i^*$  to have fast access to its surrogate  $\mathbf{n}[i^*]$  when needed for the Phong's illumination model.

# Stereo surface rendering

Finally, given two viewing angles  $\alpha$  (tilt) and  $\beta$  (spin). One can create a stereo rendition  $\hat{J} = (D_J, \mathbf{J})$  as follows.

- The view of the **left eye** using  $(\alpha, \beta - 5)$  is assigned to the **red** component of  $\mathbf{J}$ .
- The view of the **right eye** using  $(\alpha, \beta + 5)$  is assigned to the **blue** component of  $\mathbf{J}$ .
- The **average** between the two views is assigned to the **green** component of  $\mathbf{J}$ .

