Image Processing using Graphs (lecture 3 - image segmentation)

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• Delineation was solved by spel classification, so connectivity was not needed.

When do we need connectivity?

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However, when do we need optimum connectivity?

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Optimum connectivity is needed for delineation when object and parts of the background with similar properties are connected to each other.



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In this case, however, the markers needed to disconnect the object are not suitable for classification.

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 fuzzy spel classification for arc-weight estimation in IFT-based segmentation methods,

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- fuzzy spel classification for arc-weight estimation in IFT-based segmentation methods,
- a synergism between recognition by a human operator and IFT-based delineation in interactive segmentation, and

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- fuzzy spel classification for arc-weight estimation in IFT-based segmentation methods,
- a synergism between recognition by a human operator and IFT-based delineation in interactive segmentation, and
- a synergism between recognition by object models and IFT-based delineation in automatic segmentation.

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• Arc-weight estimation [1, 2].



- Arc-weight estimation [1, 2].
- Boundary-based delineation [3].



- Arc-weight estimation [1, 2].
- Boundary-based delineation [3].
- Region-based delineation [4, 5].



- Arc-weight estimation [1, 2].
- Boundary-based delineation [3].
- Region-based delineation [4, 5].
- Cloud system model (CSM) [6].

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For a given image $I = (D_I, \vec{I})$ and adjacency relation \mathcal{A} (e.g., 8-neighbors in 2D and 6-neighbors in 3D).

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• The success of segmentation strongly depends on the arc-weight estimation, which affects path function $f(\pi_t)$.

For a given image $\mathbf{I} = (\mathcal{D}_I, \vec{I})$ and adjacency relation \mathcal{A} (e.g., 8-neighbors in 2D and 6-neighbors in 3D).

- The success of segmentation strongly depends on the arc-weight estimation, which affects path function $f(\pi_t)$.
- The weight $0 \le w(s,t) \le K$ of an arc $(s,t) \in A$ can be a linear combination

$$w(s,t) = \alpha w_o(s,t) + (1-\alpha)w_i(s,t),$$

where

- $w_o(s, t)$ takes into account object information (e.g., its features, markers, model), and
- $w_i(s, t)$ takes into account local image features.
- $0 \le \alpha \le 1$ gives the importance of each component.

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Arc weights can be visualized by a weight image $\mathbf{W} = (\mathcal{D}_I, W)$:

$$W(s) = \max_{\forall t \in \mathcal{A}(s)} \{w(s, t)\}$$





using $w_i(s, t)$ only, for $\alpha = 0.0$.

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using $w_o(s, t)$ and $w_i(s, t)$, for $\alpha = 0.8$.

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For interactive segmentation of natural images:

- Object-based weights $w_o(s, t)$ can be derived from
 - color image features,
 - user-drawn markers, and
 - fuzzy spel classification.

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 - color image features,
 - user-drawn markers, and
 - fuzzy spel classification.
- Image-based weights w_i(s, t) use only the local values of the same image features.
- Markers used for IFT delineation are never used for arc-weight estimation.

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We may

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- use connected filters to reduce noise,
- convert image $\mathbf{I} = (\mathcal{D}_I, \vec{I})$ from the RGB to the Lab color space, creating a feature image $\mathbf{F} = (\mathcal{D}_I, \vec{F})$,

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We may

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) from the RGB to the Lab color space, creating a feature image F = (D_I, F
),
- create an object map $\mathbf{O} = (\mathcal{D}_I, O)$ by fuzzy classification, and
- define image-based and object-based weights by

$$egin{array}{lll} w_i(s,t) &\propto & \|ec{F}(t)-ec{F}(s)\| \ w_o(s,t) &\propto & |O(t)-O(s)|, \end{array}$$

where O(s) is higher when s is an object spel.





• For a given image.

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- For a given image.
- Training markers are drawn in distinct parts of object and background.
- Classification gives a visual feedback to guide new marker selection (synergism).
- As markers are added, classification improves the object map.
• boundary-based delineation by live-wire-on-the-fly [3],

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- boundary-based delineation by live-wire-on-the-fly [3],
- region-based delineation using the differential IFT algorithm with seed competition (IFTSC) [4], and

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- boundary-based delineation by live-wire-on-the-fly [3],
- region-based delineation using the differential IFT algorithm with seed competition (IFTSC) [4], and
- a comparative analysis between IFTSC and segmentation based on the min-cut/max-flow algorithm [5].

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Optimum paths are incrementally computed from the wavefront Q.

• The user selects a point A on the object's boundary, and





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- for any subsequent position of the cursor, an optimum path from A to that position is displayed in real time.
- When the cursor is close to the boundary, the path snaps on to it.
- The user may accept it as a boundary segment, and
- the process is repeated from its terminus *B* until the user decides to close the contour.

The IFT algorithm with early termination and function f_{sum} finds optimum paths from a starting point s^* on anti-clockwise oriented boundaries.

$$f_{sum}(\langle t \rangle) = \begin{cases} 0 & \text{if } t = s^* \\ +\infty & \text{otherwise} \end{cases}$$

$$f_{sum}(\pi_s \cdot \langle s, t \rangle) = \begin{cases} f_{sum}(\pi_s) + \bar{w}^{\beta}(s, t) & \text{if } O(l) \ge O(r) \\ f_{sum}(\pi_s) + K^{\beta} & \text{otherwise,} \end{cases}$$

where *I* and *r* are the spels at the left and right sides of arc $\langle s, t \rangle$. The weights $\bar{w}(s,t)$ are lower on the boundary than inside and outside it and $\beta \geq 1$ favors longer segments. (Show software)

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Path computation from s^* to t in LWOF

Algorithm

– Path Computation from s^* to t in LWOF

```
1.
    If V(t) = +\infty or t \in Q, then
2.
            While Q is not empty, do
3.
                   Remove from Q a spel s such that V(s) is minimum.
4.
                   If s = t then return.
5.
                   For each t \in A(s) such that V(t) > V(s), do
6.
                          If O(l) \geq O(r),
                                 then set tmp \leftarrow V(s) + \bar{w}^{\beta}(s, t)
7.
                                Else set tmp \leftarrow V(s) + K^{\beta}.
8.
9.
                          If tmp < V(t), then
10.
                                 If V(t) \neq +\infty, remove t from Q.
11.
                                 Set P(t) \leftarrow s and V(t) \leftarrow tmp.
12
                                 Insert t in Q.
```

Differential IFT with seed competition (IFTSC)

According to lecture 1, the object can also be defined by using the differential IFT algorithm with seed competition [4].



 Internal (A) and external (B and C) markers are selected, but a "leaking" occurs. According to lecture 1, the object can also be defined by using the differential IFT algorithm with seed competition [4].



- Internal (A) and external (B and C) markers are selected, but a "leaking" occurs.
- We add an external marker *D* and select marker *C* for removal. The competition involves *D* and frontier spels (dashed line) of the forests of *A* and *B*.

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- Internal (A) and external (B and C) markers are selected, but a "leaking" occurs.
- We add an external marker *D* and select marker *C* for removal. The competition involves *D* and frontier spels (dashed line) of the forests of *A* and *B*.
- Result of segmentation.

In this case, the object is an optimum-path forest for $f_{\rm max}$ rooted at internal seeds.

$$\begin{split} f_{\max}(\langle t \rangle) &= \begin{cases} 0 & \text{if } t \in \mathcal{S} = \mathcal{S}_i \cup \mathcal{S}_e \\ +\infty & \text{otherwise} \end{cases} \\ f_{\max}(\pi_s \cdot \langle s, t \rangle) &= \max\{f_{\max}(\pi_s), w(s, t)\}, \end{split}$$

where S_i and S_e are internal and external seed sets.

Differential IFT with f_{max} and seed competition (IFTSC)

Algorithm

- Algorithm IFTSC

1. $(V, P, L, \mathcal{F} \leftarrow \mathcal{F} \setminus \mathcal{S}) \leftarrow \text{DIFT-FORESTREMOVAL}(V, P, L, \mathcal{A}, \mathcal{R}_{\mathcal{M}}).$ While $S \neq \emptyset$, remove t from S, set $V(t) \leftarrow 0$, 2 3. L set L(t) ← λ (t), P(t) ← nil, and $\mathcal{F} \leftarrow \mathcal{F} \cup \{t\}$. 4. While $\mathcal{F} \neq \emptyset$, remove t from \mathcal{F} and insert t in Q. 5. While Q is not empty do Remove s from Q such that V(s) is minimum. 6. 7. For each $t \in \mathcal{A}(s)$, do 8. Compute tmp $\leftarrow \max\{V(s), w(s, t)\}$. 9. If tmp < V(t) or P(t) = s, then 10. If $t \in Q$, then remove t from Q. Set $P(t) \leftarrow s$, $V(t) \leftarrow tmp$, $R(t) \leftarrow R(s)$. 11. 12. Insert t in Q.

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- can handle multiple objects in sublinear time,
- is easily extended to 3D images, and
- is much faster and more robust than segmentation methods based on the min-cut/max-flow algorithm.

From lecture 1, we know that an optimum-path forest for f_{max} provides the graph cut whose minimum arc weight

$$\min_{\substack{\forall (s,t) \in \mathcal{A}, L(s) = 1, L(t) = 0}} w(s, t)$$

is maximum, considering all possible cuts between internal and external seeds [5].

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- In fact, the boundaries obtained by IFTSC are also piecewise optimum.
- That is, the above optimization holds to any part of the segmented boundary.

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IFTSC as a graph cut segmentation



The boundary segment with arc weights equal to 5 has preference over the segment with weight 4, due to the piecewise optimum property. Note that both solutions lead to the same maximum cut with minimum value 3.

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Segmentation by the min-cut/max-flow algorithm

 Methods based on the min-cut/max-flow algorithm usually aim to minimize the sum of arc weights w
^β(s, t) in the cut boundary.

$$\sum_{\substack{\forall (s,t) \in \mathcal{A} \mid \ L(s) = 1, L(t) = 0}} \bar{w}^{\beta}(s,t),$$

for $\beta \geq 1$, where lower values of β favor undesirable small cuts.

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for $\beta \geq 1,$ where lower values of β favor undesirable small cuts.

 For higher values of β, the above optimization tends to minimize the maximum arc weight w
 (s, t) in the cut boundary,

$$\max_{\forall (s,t)\in \mathcal{A} \mid L(s)=1, L(t)=0} \bar{w}(s,t),$$

which is essentially the same of maximizing the minimum arc weight w(s, t) in the cut boundary, as done by IFTSC.

A same connected component is always obtained with IFTSC, independently of seed location.



The same does not happen in the segmentation by the min-cut/max-flow algorithm, when β is not high enough.



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Therefore, the best result of the min-cut/max-flow algorithm is the one obtained by the IFTSC algorithm. Show software.

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 - requires translation, IFTSC delineation and evaluation of delineation by a criterion function, and

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 - does not require landmarks, point correspondences or deformable registration, only
 - requires translation, IFTSC delineation and evaluation of delineation by a criterion function, and
 - exploits model orientation in arc-weight estimation for f_{max} .

The cloud system model

A set of training objects is first provided by interactive IFTSC segmentation.



Each image with multiple objects forms an object system with a common reference point (e.g., the geometric center of the objects).

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 Each object system becomes a node of a complete graph, where the weight of each arc derives from the similarities between the corresponding objects in shape, size and position. Groups of object systems in which the corresponding objects have similar shapes, sizes and positions form different cloud system models, as follows.

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- The groups are found as maximal cliques in which all arc weights are higher than a threshold.

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The object systems in each group are finally translated to a same reference point and the corresponding object masks are averaged, forming a set of cloud systems.



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 - A probabilistic map (object clouds), which indicates an object uncertainty region with values strictly lower than 1 and higher than 0.

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 - A probabilistic map (object clouds), which indicates an object uncertainty region with values strictly lower than 1 and higher than 0.
 - A delineation algorithm (IFTSC), whose execution is constrained in the uncertainty region.
 - A criterion function, which assigns a score to any set of delineated objects.

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Segmentation using CSM consists of a search for the translation to the image location which produces the highest score, when the reference point of the most suitable cloud system is at that position.



show video handsearch.mpg

In MR-brain images, 3D objects are cerebellum and each cerebral hemisphere without brain stem.



We are using four 3D cloud systems created from MR-images of 40 normal subjects and a three-scale search for each, which is exaustive in the lowest resolution and local in the higher ones.

The criterion function assigns a score proportional to the mean arc weight in the graph cut, which separates all objects from the background.

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Brain structure segmentation



Each object cloud can also give us hints about the expected surface normal, which may be used to penalize arc weights w(s, t) with wrong gradient orientation across them.



Brain structure segmentation



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Alexandre Xavier Falcão Image Processing using Graphs at ASC-SP 2010





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• Arc-weight estimation can exploit model orientation and be used in other segmentation methods based on similar concepts: similarity, speed function, affinity, cost, distance, etc.

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- The IFTSC approach is certainly among the best delineation methods today.
- IFTSC with automatic seed propagation along video frames has been sucessfully used for object tracking [7].
- The cloud system model can be exploited in other applications rather than brain structure segmentation.

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- The IFT framework.
- Connected filters.
- Interactive and automatic segmentation methods.
- Shape representation and description.
- Clustering and classification.

Thanks for your attention

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