Introduction to Image Representation

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They may be divided into

- point-based,
- region-based, and
- shape-based methods.

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- A multiband image of the last convolutional layer of a Convolutional Neural Network (CNN).

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- Integral image.
- A multiband image of the last convolutional layer of a Convolutional Neural Network (CNN).
- A set of subimages (patches) extracted from detected key points.

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The output is a label image, or a hierarchy of region labels, that indicate which pixels should be used for region description.

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- An Euclidean distance map of all contours (external and internal ones).
- Multiscale skeletons of a shape.
- Convex and concave salience points.

Agenda

- Integral image
- Extracting features from integral image – the result is a point-based image representation.
- Space-frequency transforms: From Fourier to Wavelets.
- Multiscale image filtering – the result is a point-based image representation.

Integral image

1	2	2	4	1
3	4	1	5	2
2	3	3	2	4
4	1	5	4	6
6	3	2	1	3

From mathworks.com

The pixel values of the integral image $\hat{I}_{int} = (D_I, I_{int})$ of an image $\hat{I} = (D_I, I)$ are defined by

$$
\begin{aligned}
I_{int}(p) &= \sum_{\forall q \in \mathcal{A}(p)} I(q) \\
\mathcal{A}(p) &: \{q \in D_I \mid (x_q \leq x_p) \text{ and } (y_q \leq y_p)\}
\end{aligned}
$$

Extracting features from integral image

Recall that the Sobel-vertical-edge kernel can enhance the characters of a car plate and the integral image can be exploited to assign higher scores to the best candidate locations.

Extracting features from integral image

1	3	5	9	10
4	10 p_1	13	22 p_2	25
6	15	21	32	39
10	20 p_3	31	46 p_4	59
16	29	42	58	74

The integral value within any rectangular region $\mathcal{R}$, delimited by pixels p_1, p_2, p_3 and p_4, is

$$\sum_{\forall p \in \mathcal{R}} I(p) = I_{int}(p_4) - I_{int}(p_2) - I_{int}(p_3) + I_{int}(p_1).$$

This corresponds to the convolution between the image and an unitary kernel with adjacency defined by $\mathcal{R}$ with respect to some origin p.

Extracting features from integral image

By defining $\mathcal{R}$ around each pixel $p \in D_I$, the integral of the edge-enhanced image can be used to define candidates for the plate location by thresholding (i.e., a weak classifier) and connected component analysis.

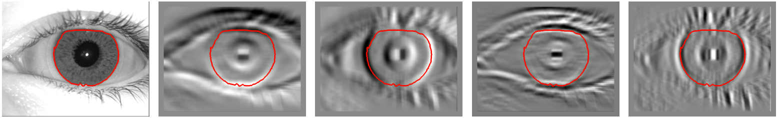
Extracting features from integral image

One may define kernels of different sizes and configurations based on integral images (haar-like features)— the weights are $w \geq 1$ in the white region(s) and $-w$ in the black region(s) — or the other way around.

Extracting features from integral image

The convolution between an image and a bank of kernels generates a multiband image $\hat{J} = (D_J, \mathbf{J})$ for feature selection and classification. Viola & Jones introduced a fast scheme based on cascade of weak classifiers for face detection [1].

Extracting features from integral image



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Space-frequency transforms

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- Let $T(u, v) = \sum_{y=0}^{n_y-1} \sum_{x=0}^{n_x-1} I(x, y) G(x, y, u, v)$, where n_x and n_y are the number of pixels along x and y, be a general transform from the coordinates $(x, y) \in D_I$ to coordinates $(u, v) \in D_T$.

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- Such functions usually allow the inverse transform $I(x,y) = \sum_{v=0}^{n_v-1} \sum_{u=0}^{n_u-1} T(u,v) H(x,y,u,v)$, such that $n_u = n_x$ and $n_v = n_y$.

Space-frequency transforms

- The most popular example is the discrete Fourier Transform.

$$
\begin{aligned}
G(x,y,u,v) &= \exp\left[-j2\pi\left(\frac{ux}{n_x}+\frac{vy}{n_y}\right)\right],\\
H(x,y,u,v) &= \frac{1}{n_x n_y}\exp\left[j2\pi\left(\frac{ux}{n_x}+\frac{vy}{n_y}\right)\right],
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- Such general transforms can be applied to any nD function, such as the image of an object, the $x(s)$ and $y(s)$ coordinates of its external contour for $s = 0, 1, \ldots, n-1$, etc.

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- The coefficients $T(u,v)$ can then be used for image/shape description.

It is also common that $G(x,y,u,v)$ and $H(x,y,u,v)$ are separable and symmetric,

$$\begin{aligned} G(x,y,u,v) &= G'(x,u)G'(y,v), \\ H(x,y,u,v) &= H'(x,u)H'(y,v), \end{aligned}$$

which allows to decompose nD forward (and inverse) transforms into 1D transforms.

$$\begin{aligned} T(u,v) &= \sum_{y=0}^{n_y-1}\sum_{x=0}^{n_x-1} I(x,y)G(x,y,u,v) \\ T(u,v) &= \sum_{y=0}^{n_y-1}\left[\sum_{x=0}^{n_x-1} I(x,y)G'(x,u)\right]G'(y,v) \\ T(u,v) &= \sum_{y=0}^{n_y-1} T(u,y)G'(y,v) \end{aligned}$$

Examples of transforms

For example, for $n_x = 2^n$ and $b_k(z)$ being the k-th bit in the binary representation of z, we have:

- The Walsh Transform

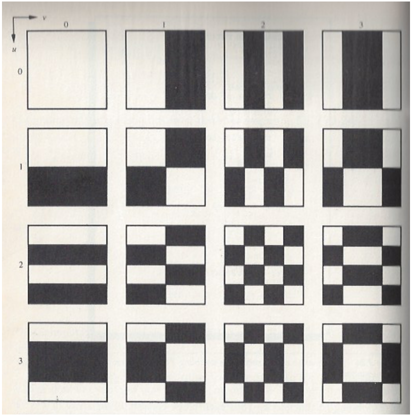
$$
\begin{aligned}
G'(x,u) &= \prod_{i=0}^{n-1} (-1)^{b_i(x) b_{n-1-i}(u)}, \\
H'(x,u) &= \frac{1}{n_x} \prod_{i=0}^{n-1} (-1)^{b_i(x) b_{n-1-i}(u)}.
\end{aligned}
$$

- The Hadamard transform

$$
\begin{aligned}
G'(x,u) &= (-1)^{\sum_{i=0}^{n-1} b_i(x) b_i(u)}, \\
H'(x,u) &= \frac{1}{n_x} (-1)^{\sum_{i=0}^{n-1} b_i(x) b_i(u)},
\end{aligned}
$$

where the summation in the exponent is performed in mod 2.

Example of Walsh basis functions



Walsh basis functions $G(x, y, 0, 0)$, $G(x, y, 0, 1)$, ..., $G(x, y, 0, 3)$, $G(x, y, 1, 0)$, $G(x, y, 1, 1)$, ..., $G(x, y, 3, 3)$, where $x, y \in \{0, 1, 2, 3\}$, white denotes $+1$, and black denotes -1 [2].

Such discrete transforms stem from the Fourier series (Joseph Fourier, 1807) for periodic and continuous signals $I(x)$ and then from the Fourier transform (1822) for aperiodic signals.

$$T(u) = \int_{-\infty}^{+\infty} I(x) G^*(x) dx,$$

where $G^*(x)$ is the complex conjugate of $G(x) = \exp[j2\pi ux]$, assuming that $\int_{-\infty}^{+\infty} |I(x)|^2 dx < +\infty$.

In order to analyze frequencies u within Gaussian windows of the space (time) domain around locations $x = \tau$, Dennis Gabor (1946) proposed the short-time Fourier Transform.

$$T(u) = \int_{-\infty}^{+\infty} I(x) G_\tau^*(x) dx,$$

where $G_\tau(x) = g(x - \tau) \exp[j2\pi ux]$ and $g(x - \tau)$ is a Gaussian window around $x = \tau$.

In order to analyze frequencies u within Gaussian windows for distinct locations and scales, one can use, for instance, Morlet (Gabor) wavelet transform (Morlet, 1970 and Grossmann and Morlet, 1984).

$$T(u) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} I(x) G^{*}_{\tau,s}(x) dx,$$

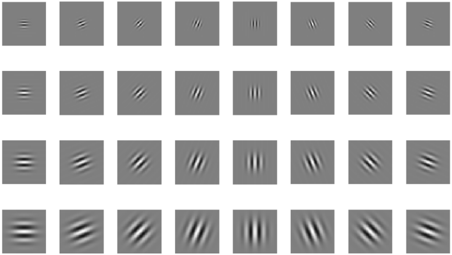
where s is the scale and

$$G_{\tau,s}(x) = \exp\left[-\pi \frac{(x-\tau)^2}{s^2}\right] \exp\left[j 2\pi u (x-\tau)\right],$$

being the mother wavelet defined by $\tau = 0$ and $s = 1$.

In 2D, the basins functions (Gabor wavelets) become

$$G_{\tau_x,\tau_y,s_x,s_y}(x,y) = \exp\left\{-\pi\left[\frac{(x-\tau_x)^2}{s_x^2}+\frac{(y-\tau_y)^2}{s_y^2}\right]\right\} \exp\left\{j2\pi\left[u\left(x-\tau_x\right)+v\left(y-\tau_y\right)\right]\right\}.$$



The real part of the basis functions for distinct orientations and scales.

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Multiscale Image Filtering

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- One can use a filter bank with Gabor wavelets, in multiple scales and orientations, as kernels for linear filtering (feature extraction).

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- Likewise, kernel banks can be composed by other basis functions (e.g., the first wavelets by Alfred Haar, 1909).

[1] P. Viola and M. Jones.
Rapid object detection using a boosted cascade of simple features.
In *Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition. CVPR 2001*, volume 1, pages I–511–I–518 vol.1, 2001.

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