# Image Description: Histogram of Oriented Gradients 

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- The extension to color images can simply concatenate the HoG feature vectors of each band inside the window.


## Agenda

The Histogram of Oriented Gradients (HoG) is a texture descriptor, which consists of the following steps.

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- Intensity normalization, gradient computation, and window definition.
- Cell definition.
- HoG computation per cell and pixel votes.
- Vote distribution.
- Coding - feature vector definition.


## Intensity normalization and gradient computation

- As first step, the image intensities are normalized within an interval $[0-L]$ (e.g., by gamma correction).

$$
\begin{gathered}
I^{\prime}(p)=K\left[\frac{I(p)}{I_{\max }}\right]^{\gamma} \\
\text { where } I_{\max }=\max _{\forall p \in D_{I}}\{I(p)\}, \gamma>0 \text {, and } K=2^{b}-1 .
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- Now, for each window of size $n_{1} \times m_{1}$ pixels around a candidate object, the HoG feature vector requires the estimation of a gradient vector $\vec{g}(p)$ at each pixel $p$.

$$
\vec{g}(p)=\sum_{\forall q \in \mathcal{A}_{r}(p)}[I(q)-I(p)] \exp \left(-\frac{\|q-p\|^{2}}{2 \sigma^{2}}\right) \overrightarrow{p q},
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where $\sigma=r / 3, \overrightarrow{p q}=\frac{q-p}{\|q-p\|}$ and $r \geq 1$.

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- The magnitude $\|\vec{g}(p)\|$ and orientation $\theta(p)$ (angle between $\vec{g}(p)$ and $x$ ) are used as follows.


## Cell definition

The window is further divided into an integer number of cells containing $n_{2} \times m_{2}$ pixels each.


## HoG computation per cell and pixel votes

- One histogram of gradient orientations per cell is obtained with $n_{b}$ bins.
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- For $n_{b}=9$ bins, for instance, the bin 0 may be used to accumulate votes from pixels whose $\|\vec{g}(p)\|=0$ and the remaining bins store votes from pixels whose $\theta(p)$ falls within [ $0-44$ ], [45-89], .., [315-359], respectively.
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- The orientation $\theta(p)$ for $h_{x}(p)=\frac{g_{x}(p)}{\|\vec{g}(p)\|}$ and $h_{y}(p)=\frac{g_{y}(p)}{\|\vec{g}(p)\|}$ is defined as

$$
\theta(p)= \begin{cases}\frac{180}{\pi} \cos ^{-1}\left(h_{x}(p)\right) & \text { if } h_{y}(p) \geq 0 \\ 360-\frac{180}{\pi} \cos ^{-1}\left(h_{x}(p)\right) & \text { if } h_{y}(p)<0\end{cases}
$$

## Vote distribution

- Each pixel $p$ distributes $\|\vec{g}(p)\|$ votes by trilinear interpolation between adjacent bins $b_{1}$ and $b_{2}$ of its four adjacent cells $q_{1}, q_{2}, q_{3}$, and $q_{4}$.

Window



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Window



- For $\theta=30$, for instance, $b_{1}=22$ and $b_{2}=67$, since the center of the 8 bins with non-zero gradient magnitude are represented by $22,67,112,157,202,247,292$, and 337.


## Vote distribution

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- Let $\left(x_{p}, y_{p}, z_{p}\right), z_{p}=\theta(p)$, be the coordinate of $p$ in a 3D space.
- Let $\left(x_{i}, y_{i}\right)$ be the center of the cell $q_{i}, i=1,2,3,4$ and $\left(q_{1}, b_{1}\right),\left(q_{2}, b_{1}\right),\left(q_{3}, b_{1}\right),\left(q_{4}, b_{1}\right),\left(q_{1}, b_{2}\right),\left(q_{2}, b_{2}\right),\left(q_{3}, b_{2}\right)$, and $\left(q_{4}, b_{2}\right)$ be the 8 vertices $\left(x_{i}, y_{i}, z_{i}\right), i=1,2, \ldots, 8$, around $p$.


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- The gradient magnitude $w=\|\vec{g}(p)\|$ is a weight distributed among the 8 vertices by trilinear interpolation.


## Vote distribution

The weight $w=\|\vec{g}(p)\|$ is first distributed between points $p_{1}$ and $p_{2}$ on opposite faces, then the weights on the faces are distributed among points $p_{3}, p_{4}, p_{5}, p_{6}$ of opposite edges, and finally the edge weights are distributed to the vertices $p_{7}, p_{8}, p_{9}, p_{10}, p_{11}, p_{12}, p_{13}$, and $p_{14}$ of the corresponding edges.


## Vote distribution

The weights $w_{i}$ of each point $p_{i}=\left(x_{p_{i}}, y_{p_{i}}, z_{p_{i}}\right), i=1,2, \ldots, 14$, are computed as

$$
\begin{aligned}
& w_{1}=w \frac{\left(x_{p_{2}}-x_{p}\right)}{\left(x_{p_{2}}-x_{p_{1}}\right)} \\
& w_{2}=w \frac{\left(x_{p}-x_{p_{1}}\right)}{\left(x_{p_{2}}-x_{p_{1}}\right)} \\
& w_{3}=w_{1} \frac{\left(y_{p_{1}}-y_{p_{4}}\right)}{\left(y_{p_{3}}-y_{p_{4}}\right)} \\
& w_{4}=w_{1} \frac{\left(y_{p_{3}}-y_{p_{1}}\right)}{\left(y_{p_{3}}-y_{p_{4}}\right)}
\end{aligned}
$$

## Vote distribution

$$
\begin{aligned}
& w_{5}=w_{2} \frac{\left(y_{p_{2}}-y_{p_{6}}\right)}{\left(y_{p_{5}}-y_{p_{6}}\right)} \\
& w_{6}=w_{2} \frac{\left(y_{p_{5}}-y_{p_{2}}\right)}{\left(y_{p_{5}}-y_{p_{6}}\right)} \\
& w_{7}=w_{3} \frac{\left(z_{p_{11}}-z_{p_{3}}\right)}{\left(z_{p_{11}}-z_{p_{7}}\right)} \\
& w_{11}=w_{3} \frac{\left(z_{p_{3}}-z_{p_{7}}\right)}{\left(y_{p_{11}}-z_{p_{7}}\right)} \\
& w_{8}=w_{4} \frac{\left(z_{p_{12}}-z_{p_{4}}\right)}{\left(z_{p_{12}}-z_{p_{8}}\right)} \\
& w_{12}=w_{4} \frac{\left(z_{p_{4}}-z_{p_{8}}\right)}{\left(z_{p_{12}}-z_{p_{8}}\right)}
\end{aligned}
$$

## Vote distribution

$$
\begin{aligned}
w_{10} & =w_{5} \frac{\left(z_{p_{14}}-z_{p_{5}}\right)}{\left(z_{p_{14}}-z_{p_{10}}\right)} \\
w_{14} & =w_{5} \frac{\left(z_{p_{5}}-z_{p_{10}}\right)}{\left(z_{p_{14}}-z_{p_{10}}\right)} \\
w_{9} & =w_{6} \frac{\left(z_{p_{13}}-z_{p_{6}}\right)}{\left(z_{p_{13}}-z_{p_{9}}\right)} \\
w_{13} & =w_{6} \frac{\left(z_{p_{6}}-z_{p_{9}}\right)}{\left(z_{p_{13}}-z_{p_{9}}\right)}
\end{aligned}
$$

Finally the weights $w_{i}$ are accumulated in the corresponding bin of the cell represented by $p_{i}, i=7,8,9,10,11,12,13,14$.

## Coding: feature vector definition

- Now, each group of $n_{3} \times m_{3}$ cells constitutes a block.


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- Adjacent blocks are defined by stride (displacement in $x$ and $y)$.

Window
Block of $2 \times 2$ cells


## Coding: feature vector definition

The cell histograms in each block are concatenated from left to right, top to bottom, and normalized, to treat contrast variations. Similarly, the block feature vectors are concatenated to output a HoG feature vector for the window.


Histogram of Oriented Gradients computed in each cell


## Coding: feature vector definition

- Let $h_{k}(i), i=0,1, \ldots, n_{b}-1$ and $k=1,2, \ldots, n_{3} \times m_{3}$, be the cell histograms in a block with $n_{3} \times m_{3}$ cells.


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- Their concatenation from left to right, top to bottom, generates a vector with features $v_{j}, j=1,2, \ldots, n_{b} \times n_{3} \times m_{3}$.
- These features are normalized as

$$
v_{j}=\frac{v_{j}}{\sqrt{\sum_{j=1}^{n_{b} \times n_{3} \times m_{3}} v_{j} v_{j}}+\epsilon}
$$

where $\epsilon$ is a small number.

## Example

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- If each block is defined by $2 \times 2$ cells and the stride is 1 cell in $x$ and $y$, each window generates $20 \times 5$ blocks.
- The four cell histograms of 9 bins in each block are concatenated and normalized to compose a vector of 36 features per block.


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- If each block is defined by $2 \times 2$ cells and the stride is 1 cell in $x$ and $y$, each window generates $20 \times 5$ blocks.
- The four cell histograms of 9 bins in each block are concatenated and normalized to compose a vector of 36 features per block.
- The feature vectors of the blocks are then concatenated to form a HoG vector with $20 \times 5 \times 36$ features.

