Fundamentals of Image Processing (part I)

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Fundamentals of Image Processing

Image processing [1] is the area of Computer Science that transforms images from one representation to another for

- quality enhancement,
- geometric transformations,
- reconstruction from projections,
- coding (compression), decoding,
- representation, description,
- segmentation, ...

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- adjacency-based transformations: the value of a pixel p in the ouput image depends on the values of its adjacent pixels in the input image.
- connectivity-based transformations: the value of a pixel p in the ouput image depends on the values of a sequence of adjacent pixels in the input image(s) with terminus at p.

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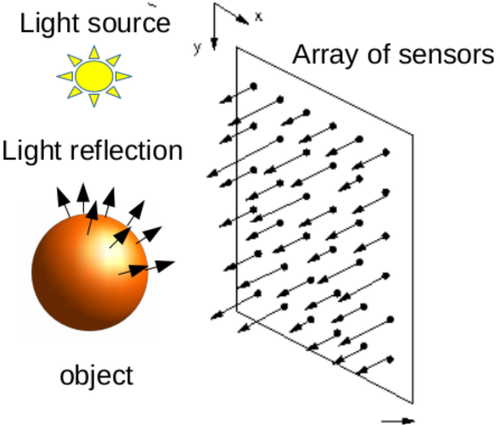
The initial lectures will provide concepts and examples from the above categories that are important for this course.

Agenda

- Digital Images: definition and representation.
- Point-based transformations.
- Adjacency relations and simple morphological operations.

Gray-scale image

A gray-scale image may be acquired from an array of sensors that measure light reflection on visible object points (sampling), transforming those measures into integer numbers (quantization).



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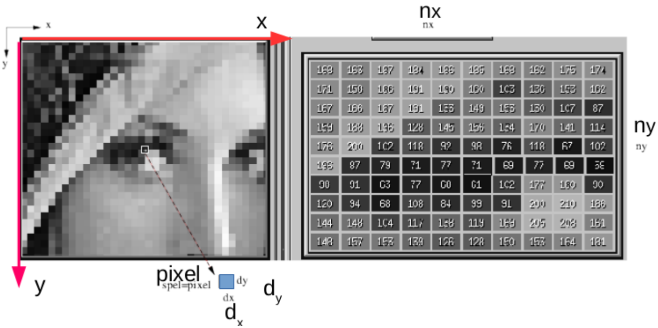


Sampling and quantization

- Sampling: A continuous function $f(x, y)$ of light reflection is measured on the 2D array of sensors at $p = (x, y)$ positions (pixels) separated by (d_x, d_y) (spatial resolution) to form a discrete function $I(x, y)$.

$$
\begin{aligned}
I(x, y) &= f(x, y) \sum_{j=0}^{n_y - 1} \sum_{i=0}^{n_x - 1} \delta(x - id_x, y - jd_y) \\
\delta(x - id_x, y - jd_y) &= \begin{cases} 1 & \text{if } x = id_x \text{ and } y = jd_y, \\ 0 & \text{otherwise.} \end{cases}
\end{aligned}
$$

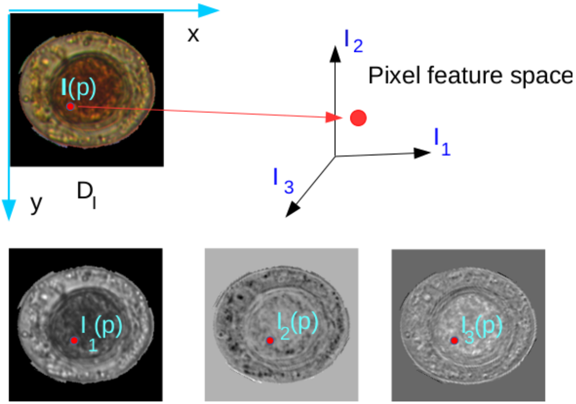
- Quantization: The values $I(x, y)$ are quantitized into integer values $I(p) \in [0, 2^b - 1]$ of b bits (image depth).

Digital image

- Images may be acquired using multiple filters of distinct wavelengths, which creates a grayscale image for each channel (band).
- By image processing, the image values can also be transformed into real values.
- Formally, a 2D multi-band image $\hat{I}$ is a pair $(D_I, \mathbf{I})$ in which $\mathbf{I}(p) \in \Re^m$ assigns m scalar values to each pixel $p \in D_I \subset \mathcal{Z}^2$.
- The image domain D_I may also have more than 2 dimensions, but this is out of the scope of this course.

Digital image

For $m = 3$ bands, each pixel p is represented by a point $\mathbf{I}(p) = (I_1(p), I_2(p), I_3(p)) \in \Re^3$.

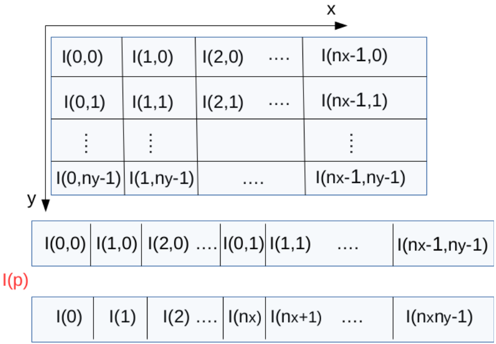


A color image with its three color components (gray-scale images).

Representation

A gray-scale image (or each band of a color image) may be stored in a 2D array $I(x_p, y_p)$ or a vector $I(p)$,

$$\begin{aligned} p &= x_p + y_p n_x, \\ x_p &= p \% n_x, \\ y_p &= \lfloor p / n_x \rfloor. \end{aligned}$$



Point-based operations

Examples of point-based operations are mathematical operations between images.

- Let $\hat{I} = (D_I, I)$ and $\hat{J} = (D_I, J)$ be two gray-scale images with the same domain and $\odot$ be any logical/arithmetical operation.

- The resulting image $\hat{K} = (D_I, K) = \hat{I} \odot \hat{J}$ is obtained by applying the operation pixel by pixel.

  - Adition and subtraction: $K(p) = I(p) + J(p)$ and $K(p) = I(p) - J(p)$.

  - Union and intersection: $K(p) = \max\{I(p), J(p)\}$ and $K(p) = \min\{I(p), J(p)\}$.

Point-based operations

Mathematical operations might also involve $\hat{I}$ and a scalar or a function to create $\hat{K}$.

- Square root $\hat{K} = \sqrt{\hat{I}}$: $K(p) = \sqrt{I(p)}$.
- Logarithm $\hat{K} = \log \hat{I}$: $K(p) = \log I(p)$.
- Module of the difference $\hat{K} = |\hat{I} - \hat{J}|$: $K(p) = |I(p) - J(p)|$.

Point-based operations

- Point-based operations may involve color space transformations for image representation.
- From RGB to YCbCr color space, an RGB image $\hat{I}$ can be transformed into a YCbCr image $\hat{K}$ pixel by pixel as follows.

$$\begin{bmatrix} K_1(p) \\ K_2(p) \\ K_3(p) \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.169 & -0.331 & 0.500 \\ 0.500 & -0.419 & -0.081 \end{bmatrix} \begin{bmatrix} I_1(p) \\ I_2(p) \\ I_3(p) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2^b}{2} \\ \frac{2^b}{2} \end{bmatrix}$$

where b is the depth of image $\hat{I}$.

Adjacency relation

An adjacency relation $\mathcal{A} \subseteq D_I \times D_I$ may be defined in the image domain and/or feature space as a binary relation.

- $\mathcal{A}_1$: $\{(p, q) \in D_I \times D_I \mid \|q - p\| \leq \alpha_1\}$,
- $\mathcal{A}_2$: $\{(p, q) \in D_I \times D_I \mid \|\mathbf{I}(q) - \mathbf{I}(p)\| \leq \alpha_2\}$,
- $\mathcal{A}_3$: $\{(p, q) \in D_I \times D_I \mid \|q - p\| \leq \alpha_1 \text{ and } \|\mathbf{I}(q) - \mathbf{I}(p)\| \leq \alpha_2\}$,

$\alpha_1, \alpha_2 \in \Re^+$, such that $\mathcal{A}(p)$ is the set of pixels q adjacent to p.

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For the image on the right, what is the adjacency set of $p = (2,3)$ for $\mathcal{A}_1, \mathcal{A}_2$, and $\mathcal{A}_3$, when $\alpha_1 = \sqrt{5}$ and $\alpha_2 = 0$?

y \ x	0	1	2	3	4	5
0	0	1	1	0	3	2
1	2	1	2	3	2	1
2	1	0	2	3	2	0
3	2	1	2	0	1	1
4	2	2	1	2	1	0
5	1	0	2	1	0	1

Adjacency relation

- It is said translation-invariant when $\mathcal{A}$: $\{(p, q_k) \in D_I \times D_I \mid (x_{q_k}, y_{q_k}) - (x_p, y_p) = (dx_k, dy_k), k = 1, 2, \ldots, K\}$, where $\{(dx_k, dy_k)\}$ is a set of K displacements.

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- One can store the displacements and generate the set $\mathcal{A}(p) = \{q_k\}$, $q_k = (x_{q_k}, y_{q_k}) = (x_p + dx_k, y_p + dy_k)$, $k = 1, 2, \ldots, K$, for any $p \in D_I$.

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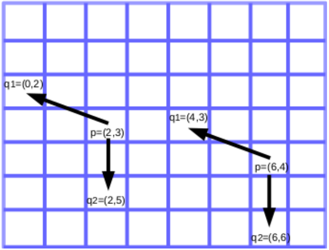
For fixed displacements $\{(-2, -1), (0, 2)\}$, examples of sets $\mathcal{A}(p) = \{q_1, q_2\}$ are



Adjacency relation

Rectangular adjacency relations $\mathcal{A}$ are very popular.

$$\mathcal{A}(p) = \{q \in D_I | \quad |x_q - x_p| \leq \frac{w}{2} \quad and \quad |y_q - y_p| \leq \frac{h}{2}\}.$$

y \ x	0	1	2	3	4	5
0	0	1	1	0	3	2
1	2	1	2	3	2	1
2	1	0	2	3	2	0
3	2	1	2	0	1	1
4	2	2	1	2	1	0
5	1	0	2	1	0	1

Which are the neighbors of $p = (2, 3)$ when $w = 3$ and $h = 5$?

Simple morphological operations

- A dilation $\hat{J} = \hat{I} \oplus \mathcal{A}$ is obtained pixel by pixel as

$$J(p) = \max_{q \in \mathcal{A}(p)} \{I(q)\}.$$

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- Compute $\hat{J} = \hat{I} \oplus \mathcal{A}$ when $\mathcal{A}$ is a 4-neighborhood relation.

I

-2	3	7	8
-8	-2	6	-9
8	1	2	7
7	9	-8	0

J

3	7	8	8
8	6	7	8
8	9	7	7
9	9	9	7

Dilation algorithm

Input: $\hat{I} = (D_I, I)$ and $\mathcal{A}$.
Output: $\hat{J} = (D_I, J)$.

1. For each $p \in D_I$, do
2. $\quad J(p) \leftarrow I(p)$.
3. $\quad$ For every $q \in \mathcal{A}(p) \setminus \{p\}$, such that $q \in D_I$, do
4. $\quad\quad$ If $I(q) > J(p)$, then
5. $\quad\quad\quad J(p) \leftarrow I(q)$.
6. Return $\hat{J}$.

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When combined with its dual operation, named erosion, they can create several types of morphological filters.

Examples

Let's see MathMorphology.ipynb in notebooks.tar.gz

[1] Rafael C. Gonzalez and Richard E. Woods.
Digital Image Processing (4th Edition).
Pearson, 2018.
www.imageprocessingplace.com.