Introduction to Lattice Based Cryptography

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Unicamp

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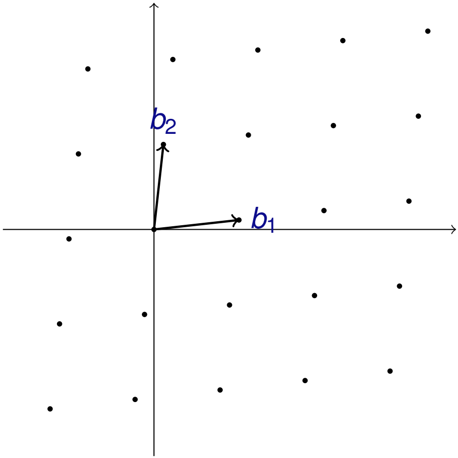
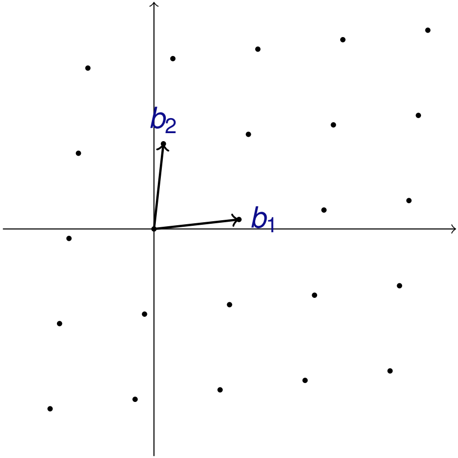
Agenda

- Introduction
  - Definitions
  - Dual Lattices
  - q-ary Lattices
  - Hard Problems
- Schemes
  - Goldreich, Goldwasser and Halevi (GGH)
  - Ajtai's construction
  - Learning With Errors (LWE), Ring LWE, NTRU-like
  - Functional Encryption, Identity Based Encryption, Attribute Based Encryption, Fully Homomorphic Encryption

Lattices



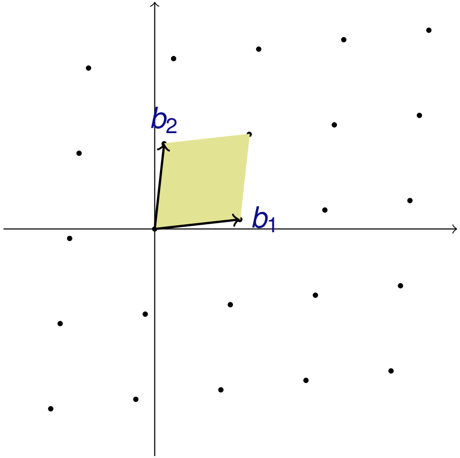
Lattices



$$\mathcal{L}(b_1, b_2) =$$

$$\{\sum x_i b_i : x_i \in \mathbb{Z}\}$$

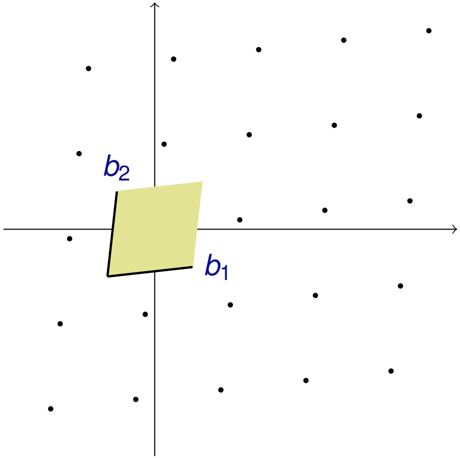
Lattices



Fundamental Domain

$$\{\sum t_i b_i, 0 \leq t_i < 1\}$$

Lattices

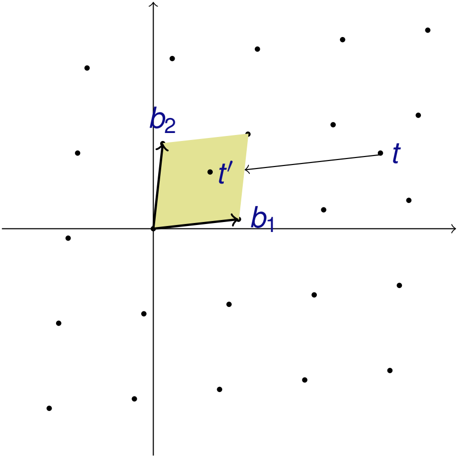


Centered

Fundamental Domain

$$\{\sum t_i b_i, \frac{-1}{2} \le t_i < \frac{1}{2}\}$$

Lattices

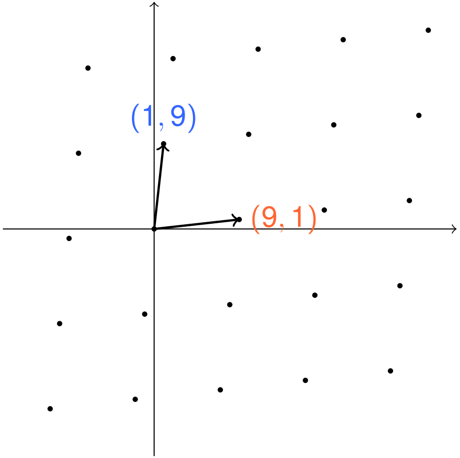


Reduction:

$$t' \equiv t \pmod{\mathcal{L}_B}$$

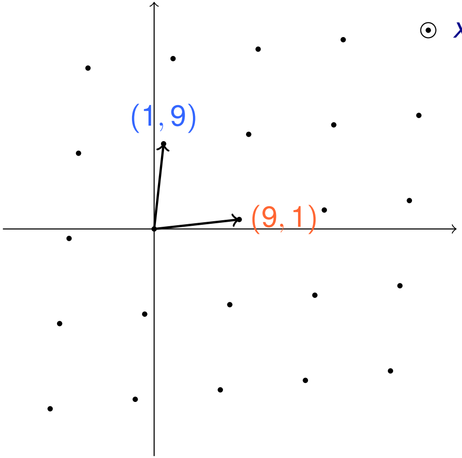
Lattices

$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



Lattices

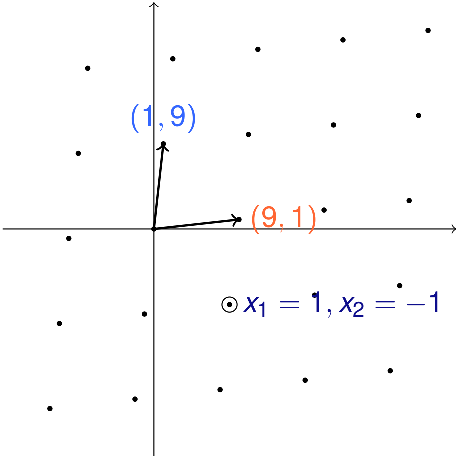
$$\mathcal{L}: \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



- $x_1 = 3, x_2 = 2$

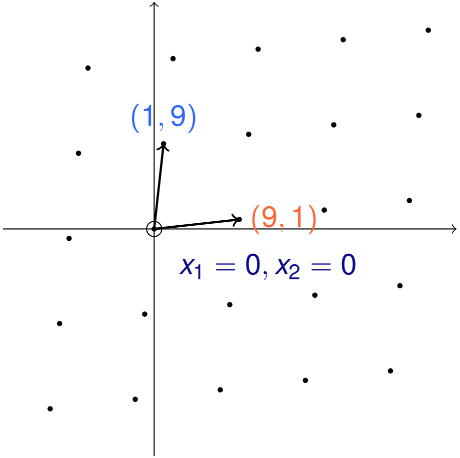
Lattices

$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



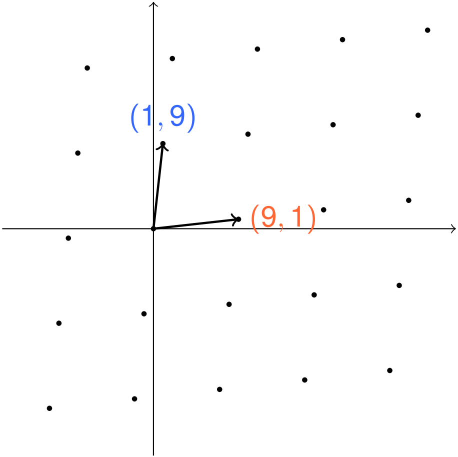
Lattices

$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



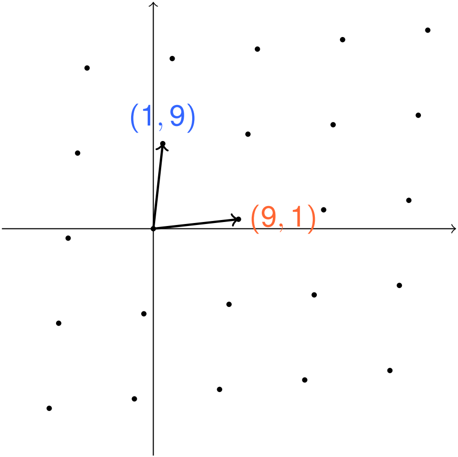
Lattices

$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



$$\begin{bmatrix} b_{1,1} & \dots & b_{n,1} \\ \vdots & \ddots & \vdots \\ b_{1,n} & \dots & b_{n,n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Lattices

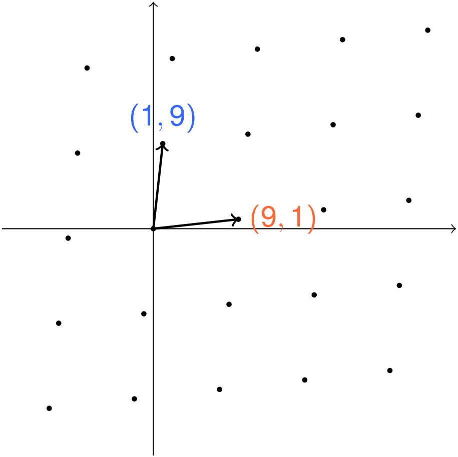


$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$Bx$$

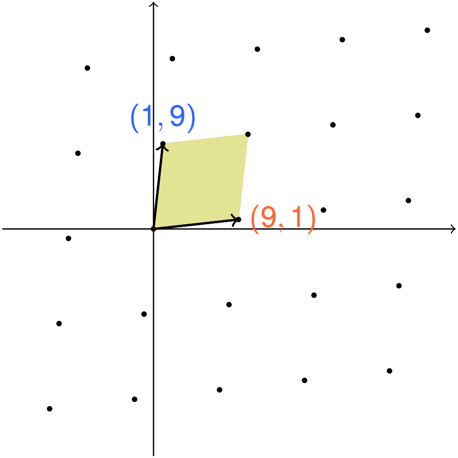
Lattices

$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



Volume of the Domain?

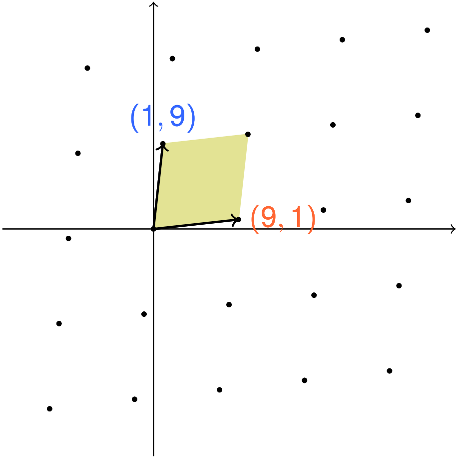
Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge:

Lattices

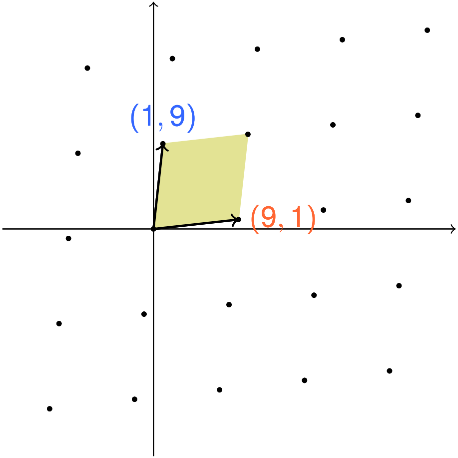


$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

$$\mathcal{A} = D.d/2$$

Lattices



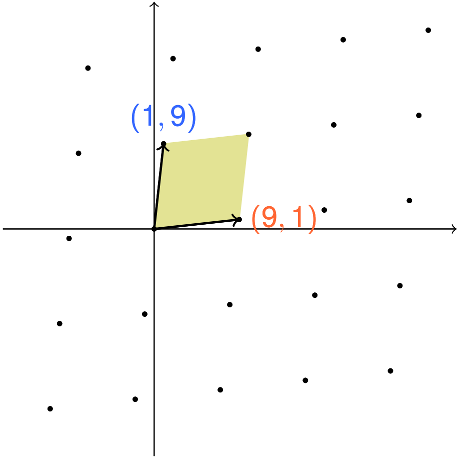
$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

$$\mathcal{A} = D.d/2$$

$$d = |(9,1) - (1,9)|$$

Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

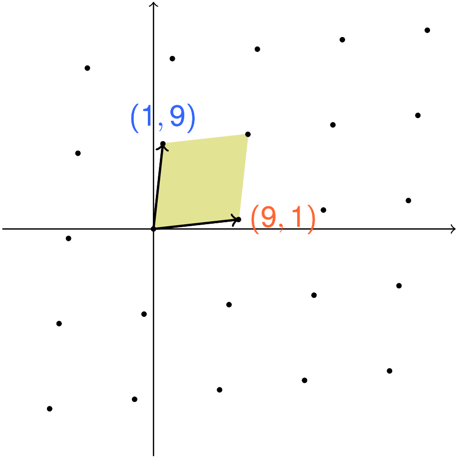
Area of lozenge

$$\mathcal{A} = D.d/2$$

$$d = |(9,1) - (1,9)|$$

$$d = \sqrt{8^2 + (-8)^2}$$

Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

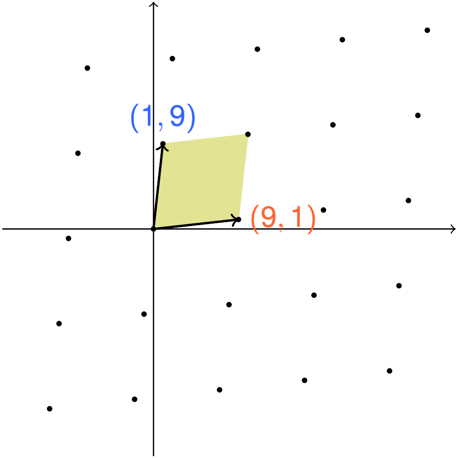
$$\mathcal{A} = D.d/2$$

$$d = |(9,1) - (1,9)|$$

$$d = \sqrt{8^2 + (-8)^2}$$

$$\boxed{d = 8\sqrt{2}}$$

Lattices



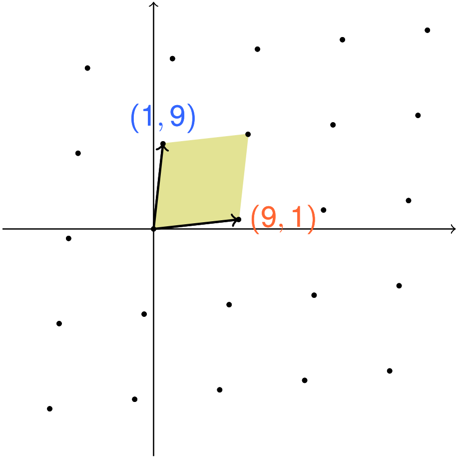
$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

$$\mathcal{A} = D.d/2$$

$$D = |(9,1) + (1,9)|$$

Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

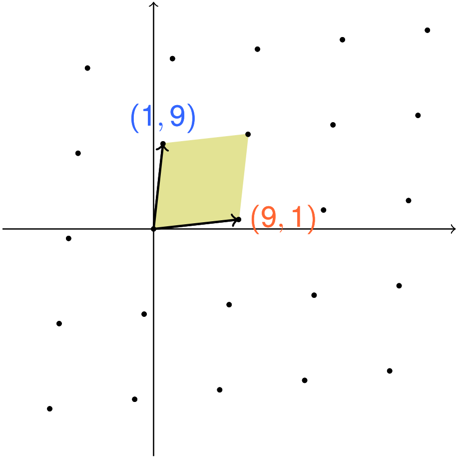
Area of lozenge

$$\mathcal{A} = D.d/2$$

$$D = |(9,1) + (1,9)|$$

$$D = \sqrt{10^2 + (-10)^2}$$

Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

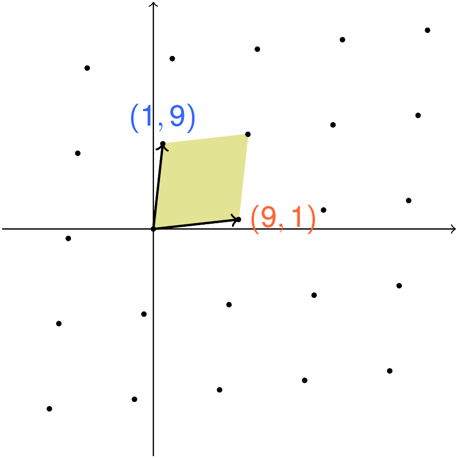
$$\mathcal{A} = D.d/2$$

$$D = |(9,1) + (1,9)|$$

$$D = \sqrt{10^2 + (-10)^2}$$

$$D = 10\sqrt{2}$$

Lattices



$$\mathcal{L}: \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

$$\mathcal{A} = D.d/2$$

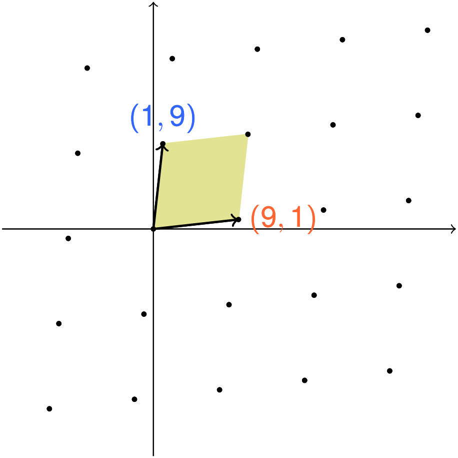
$$D = |(9,1) + (1,9)|$$

$$D = \sqrt{10^2 + (-10)^2}$$

$$D = 10\sqrt{2}$$

$$\mathcal{A} = (10\sqrt{2})(8\sqrt{2})/2$$

Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Area of lozenge

$$\mathcal{A} = D.d/2$$

$$D = |(9,1) + (1,9)|$$

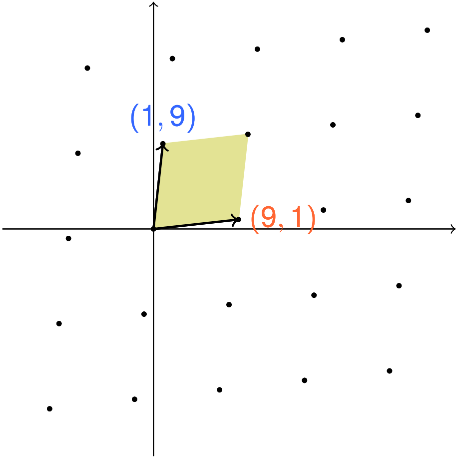
$$D = \sqrt{10^2 + (-10)^2}$$

$$D = 10\sqrt{2}$$

$$\mathcal{A} = (10\sqrt{2})(8\sqrt{2})/2$$

$$\boxed{\mathcal{A} = 10.8 = 80}$$

Lattices



$$\mathcal{L} : \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\det B = 9.9 - 1.1$$

$$\det B = 81 - 1$$

$$\det B = 80$$

$$\boxed{\text{Volume: } \det B}$$

Orthogonality

$||b_i|| \approx 9.05$

$\mathcal{A} = 80$

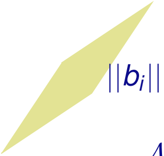
$\mathcal{A}/||b_i||^2 = 0.97$

$||b_i|| \approx 9.05$

$\mathcal{A} = 81.9$

$\mathcal{A}/||b_i||^2 = 1$

Orthogonality



$||b_i|| \approx 21.47$

$\mathcal{A} = 80$

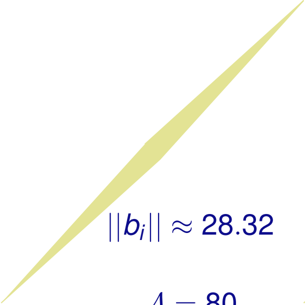
$\mathcal{A}/||b_i||^2 = 0.17$

$||b_i|| \approx 9.05$

$\mathcal{A} = 20.46$

$\mathcal{A}/||b_i||^2 = 0.125$

Orthogonality



$$||b_i|| \approx 28.32$$

$$\mathcal{A} = 80$$

$$\mathcal{A}/||b_i||^2 = 0.10$$

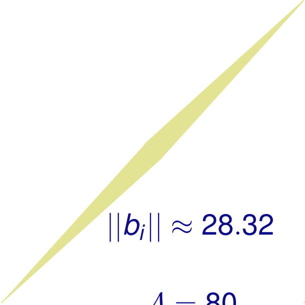
$$||b_i|| \approx 9.05$$

$$\mathcal{A} = 11.52$$

$$\mathcal{A}/||b_i||^2 = 0.14$$

Orthogonality

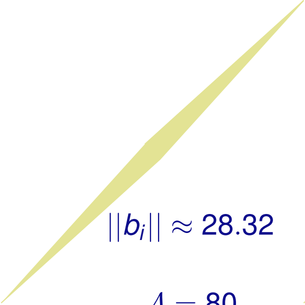
$$\boxed{\frac{\det \mathcal{L}}{\prod_{1 \le i \le n} ||b_i||}}$$



$\|\|b_i\|\| \approx 28.32$	$\|\|b_i\|\| \approx 9.05$
$\mathcal{A} = 80$	$\mathcal{A} = 11.52$
$\mathcal{A}/\|\|b_i\|\|^2 = 0.10$	$\mathcal{A}/\|\|b_i\|\|^2 = 0.14$

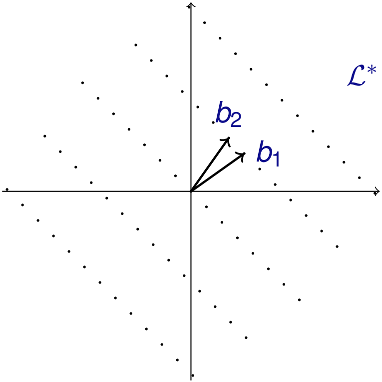
Orthogonality

$$\boxed{\left(\frac{\det \mathcal{L}}{\prod_{1\le i\le n} ||b_i||}\right)^{1/n}}$$ Hadamard Ratio



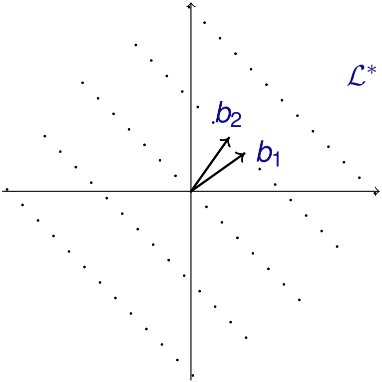
$\|\|b_i\|\| \approx 28.32$	$\|\|b_i\|\| \approx 9.05$
$\mathcal{A} = 80$	$\mathcal{A} = 11.52$
$\mathcal{A}/\|\|b_i\|\|^2 = 0.10$	$\mathcal{A}/\|\|b_i\|\|^2 = 0.14$

Dual Lattices



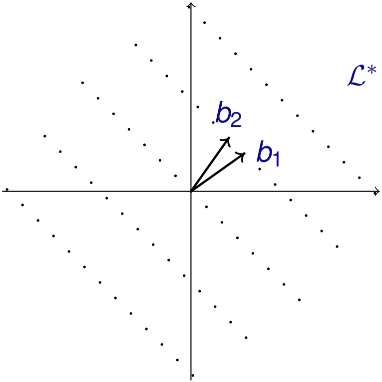
$$\mathcal{L}^* = \{y \mid \langle x, y \rangle \in \mathbb{Z}, \forall x \in \mathcal{L}\}$$

Dual Lattices



$$\mathcal{L}^* = \{y \mid \langle x, y\rangle \in \mathbb{Z}, \forall x \in \mathcal{L}\}$$
$$\langle b_1^*, b_1\rangle = 0$$
$$\langle b_1^*, b_2\rangle = 1$$

Dual Lattices



$$\mathcal{L}^* = \{y \mid \langle x, y\rangle \in \mathbb{Z}, \forall x \in \mathcal{L}\}$$

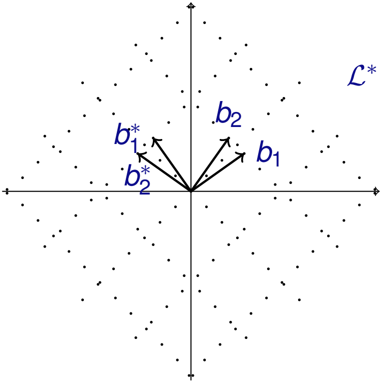
$$\langle b_1^*, b_1\rangle = 0$$

$$\langle b_1^*, b_2\rangle = 1$$

$$\langle b_2^*, b_2\rangle = 0$$

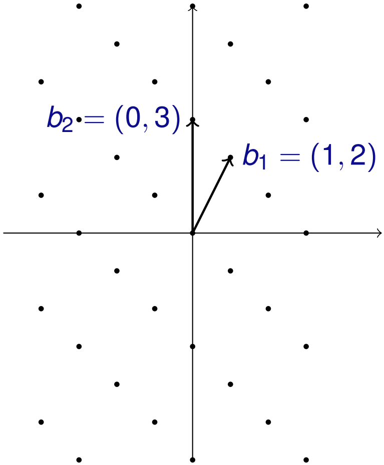
$$\langle b_2^*, b_1\rangle = 1$$

Dual Lattices

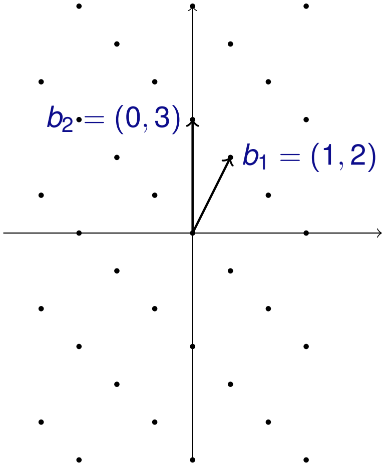


$$\mathcal{L}^* = \{y \mid \langle x, y\rangle \in \mathbb{Z}, \forall x \in \mathcal{L}\}$$
$$\langle b_1^*, b_1\rangle = 0$$
$$\langle b_1^*, b_2\rangle = 1$$
$$\langle b_2^*, b_2\rangle = 0$$
$$\langle b_2^*, b_1\rangle = 1$$

Dual Lattices



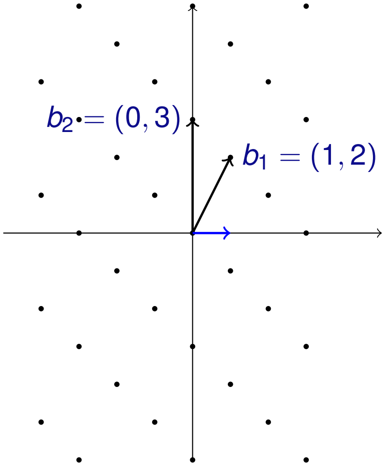
Dual Lattices



$$\langle (x_1, y_1), (0,3) \rangle = 0$$

$$\langle (x_1, y_1), (1,2) \rangle = 1$$

Dual Lattices

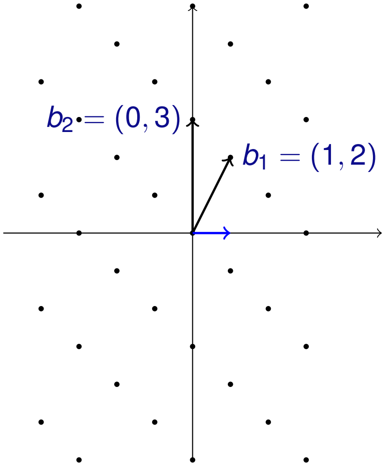


$$\langle (x_1, y_1), (0,3) \rangle = 0$$

$$\langle (x_1, y_1), (1,2) \rangle = 1$$

$$\boxed{(x_1 = 1, y_1 = 0)}$$

Dual Lattices



$$\langle (x_1, y_1), (0,3) \rangle = 0$$

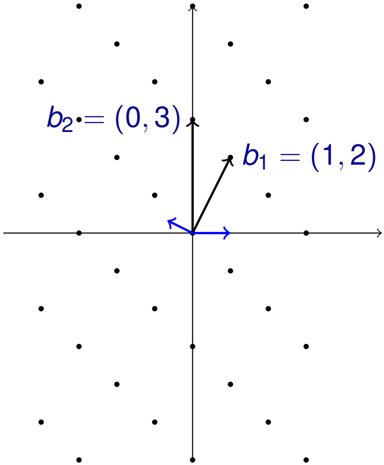
$$\langle (x_1, y_1), (1,2) \rangle = 1$$

$$\boxed{(x_1 = 1, y_1 = 0)}$$

$$\langle (x_2, y_2), (1,2) \rangle = 0$$

$$\langle (x_2, y_2), (0,3) \rangle = 1$$

Dual Lattices



$$\langle (x_1, y_1), (0,3) \rangle = 0$$

$$\langle (x_1, y_1), (1,2) \rangle = 1$$

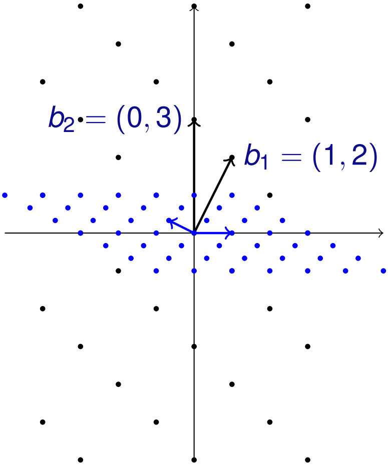
$$\boxed{(x_1 = 1, y_1 = 0)}$$

$$\langle (x_2, y_2), (1,2) \rangle = 0$$

$$\langle (x_2, y_2), (0,3) \rangle = 1$$

$$\boxed{(x_2 = -2/3, y_2 = 1/3)}$$

Dual Lattices



$$\langle (x_1, y_1), (0, 3) \rangle = 0$$

$$\langle (x_1, y_1), (1, 2) \rangle = 1$$

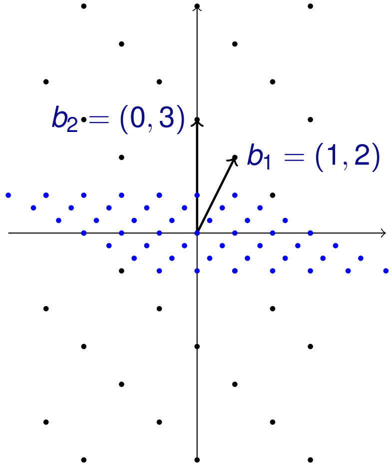
$$\boxed{(x_1 = 1, y_1 = 0)}$$

$$\langle (x_2, y_2), (1, 2) \rangle = 0$$

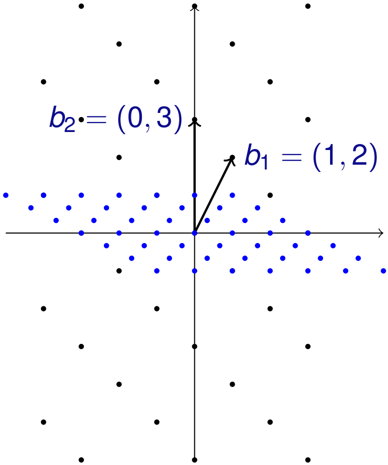
$$\langle (x_2, y_2), (0, 3) \rangle = 1$$

$$\boxed{(x_2 = -2/3, y_2 = 1/3)}$$

Dual Lattices



Dual Lattices

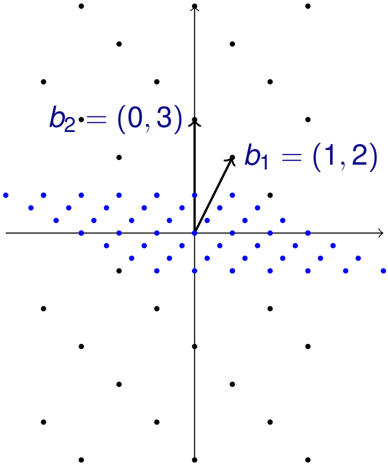


Volume:

$$\left|\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}\right| = -3$$

$$\left|\begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}\right| = 1/3$$

Dual Lattices



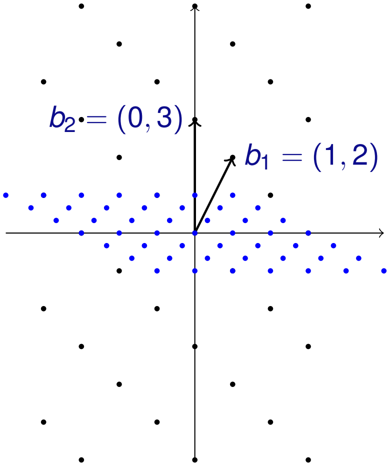
Volume:

$$\left| \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} \right| = -3$$

$$\left| \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix} \right| = 1/3$$

$$\mathcal{L}(B)^* = \mathcal{L}((B^{-1})^T)$$

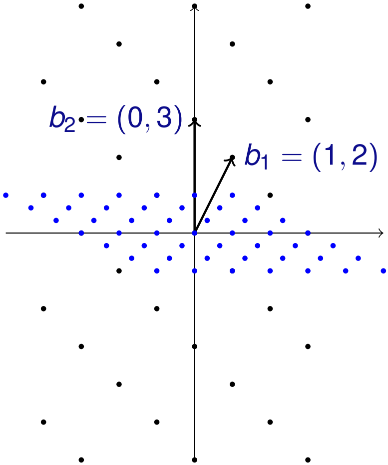
Dual Lattices



$$B^{-1} = 1/3 \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

$$(B^{-1})^T = \begin{bmatrix} -2/3 & 1 \\ 1/3 & 0 \end{bmatrix}$$

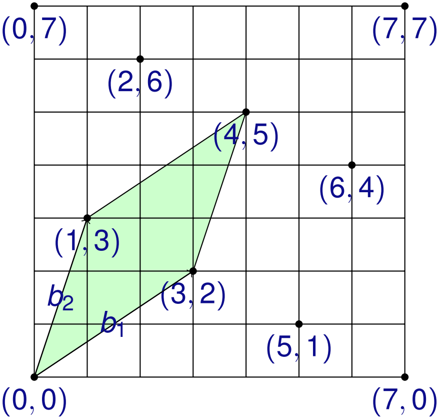
Dual Lattices



$$B^{-1} = 1/3 \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

$$(B^{-1})^T = \begin{bmatrix} 1 & -2/3 \\ 0 & 1/3 \end{bmatrix}$$

q-ary Lattices



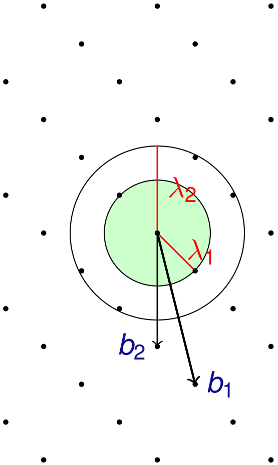
$$\Lambda_q(A) = \{y = As \pmod{q}\}$$

$$\Lambda_q^{\perp}(A) = \{y \mid Ay = 0 \pmod{q}\}$$

$$\Lambda_q^{\perp}(A) = q\Lambda_q(A)^*$$

$$\Lambda_q(A) = q\Lambda_q^{\perp}(A)^*$$

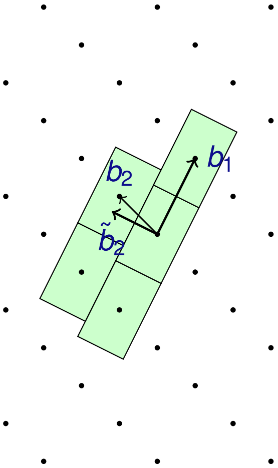
Successive Minima



λ_i: min r s.t.

$\mathcal{B}_r$ has i lin. ind. vectors

Gram-Schmidt Orthogonalization Process



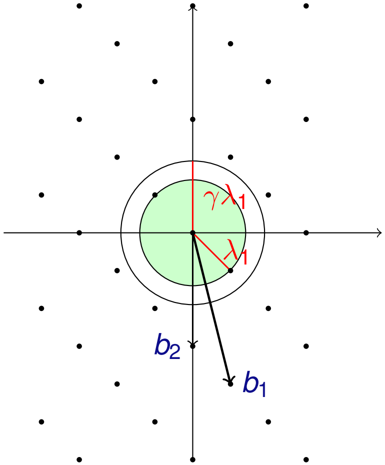
$$\tilde{B} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \mu_{2,1} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \mu_{n,1} & \dots & \mu_{n,n-1} & 1 \end{bmatrix} .B$$

$$\mu_{i,j} = \frac{\langle b_i, \tilde{b}_j \rangle}{||\tilde{b}_j||^2}$$

Minkowski's Theorem

- Pigeonhole principle for lattices
- A symmetric and convex region with volume $2^n \det(B)^{1/n}$ has at least 2 non-zero vectors
- Hermite upper bound: $\lambda_1 \leq \sqrt{n} \det(B)^{1/n}$
- Gaussian heuristics: $\lambda_1 \leq \sqrt{\frac{2n}{\pi e}} \det(B)^{1/n}$
- Lower bound: $\lambda_1 \geq \min_i ||\tilde{b}_i||$

Shortest Vector Problem (and Gap-SVP)



Search:($v \neq 0$)

$$||v|| \leq \gamma\lambda_1$$

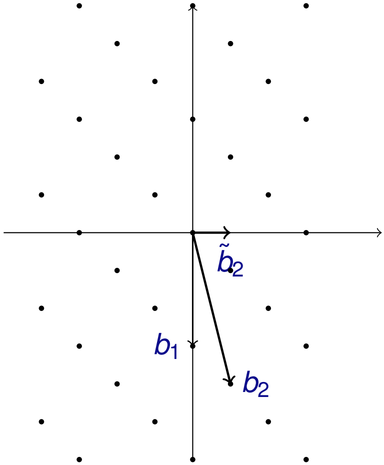
Decision: given d

$$\lambda_1 \leq \gamma d?$$

GapSVP Complexity

γ :	$2^{(\log n)^{1-\epsilon}}$	$\sqrt{n}$	n	2^n
	NP hard	NP $\cap$ coNP	crypto	P (LLL)

LLL



$$||\tilde{b}_{i+1}||^2 \geq 1/2||\tilde{b}_i||^2$$

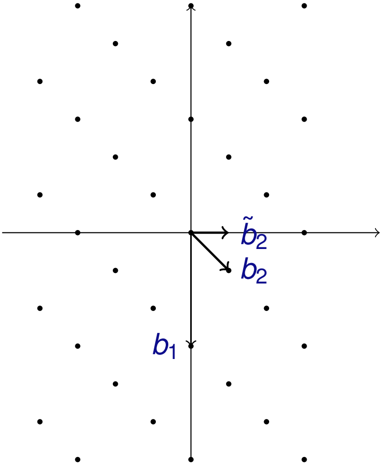
$$||b_1|| \leq 2^{(n-1)/2}\lambda_1(\mathcal{L})$$

$$b_2 = b_2 - c.b_1$$

if $||b_2||^2 < 3/4||b_1||^2$,

swap and loop

LLL



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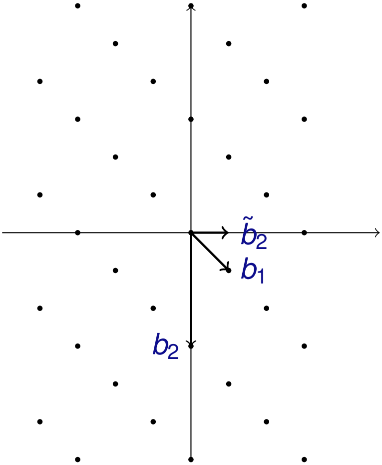

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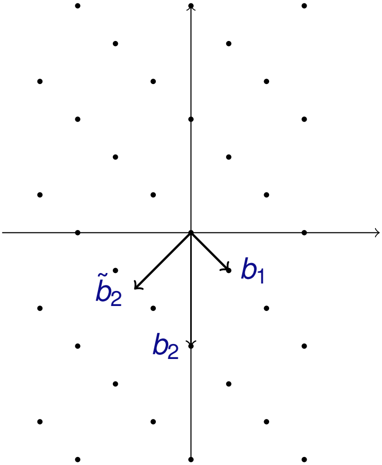

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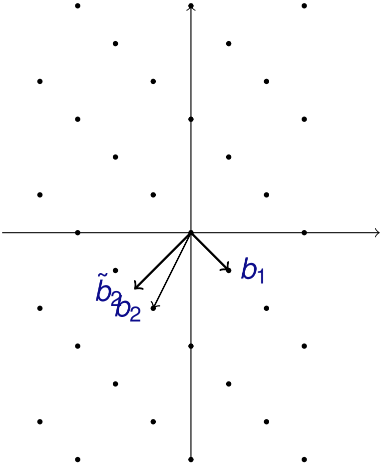

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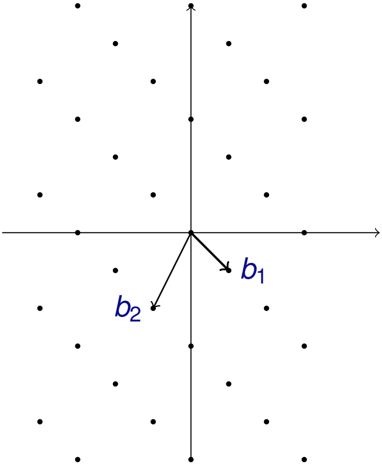

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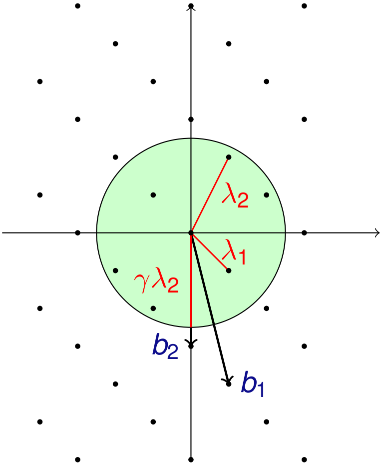
Shortest Independent Vectors Problem



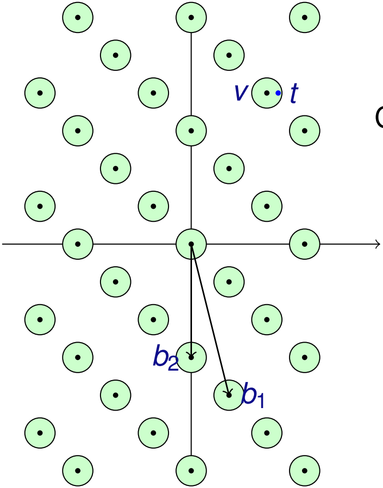
Search:

(v_1, v_2)

lin. ind.

$||v_2|| \leq \gamma\lambda_2$

Bounded Distance Decode



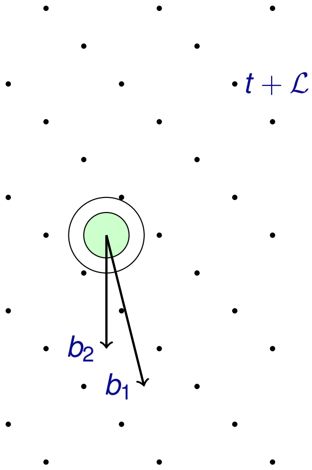
Similar to the
Closest Vector Problem (CVP)

Search: given $d < \lambda_1/2$

given $t \in \mathcal{B}_d(\mathcal{L})$

find v

Bounded Distance Decode



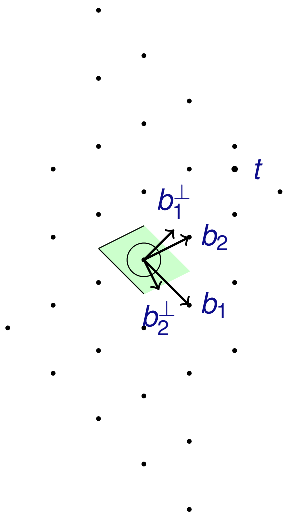
Decision: given d

given coset $t + \mathcal{L}$

decide if there is v s. t.

$||t - v|| \leq \gamma d$

Babai's Roundoff Algorithm



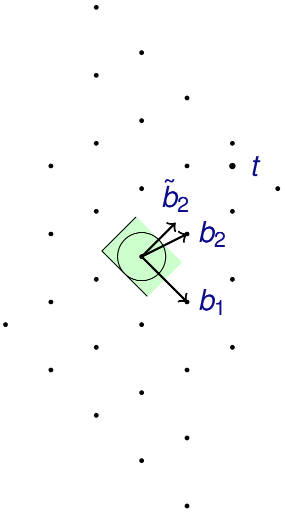
Compute $x \equiv t \pmod{B}$

$\mathcal{B}_d \subset \mathcal{P}_B$

$d = \min_i(b_i^\perp)$

(linear system)

Babai's Nearest Plane Algorithm



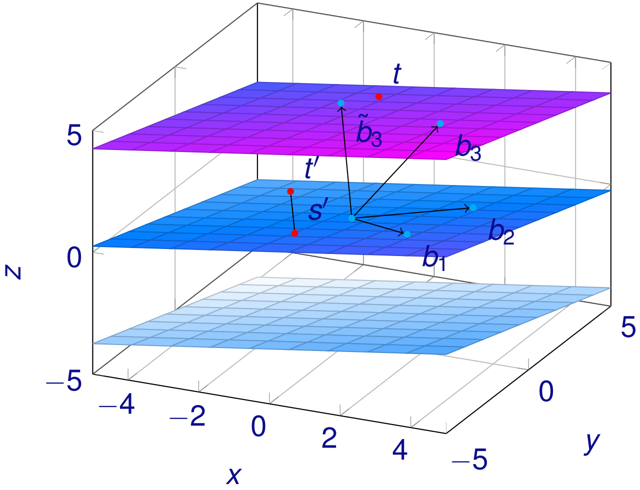
Compute $x \equiv t \pmod{\tilde{B}}$

(iteratively)

$\mathcal{B}_d \subset \mathcal{P}_{\tilde{B}}$

$d = \min_i(\tilde{b}_i)$

Babai's Nearest Plane Algorithm



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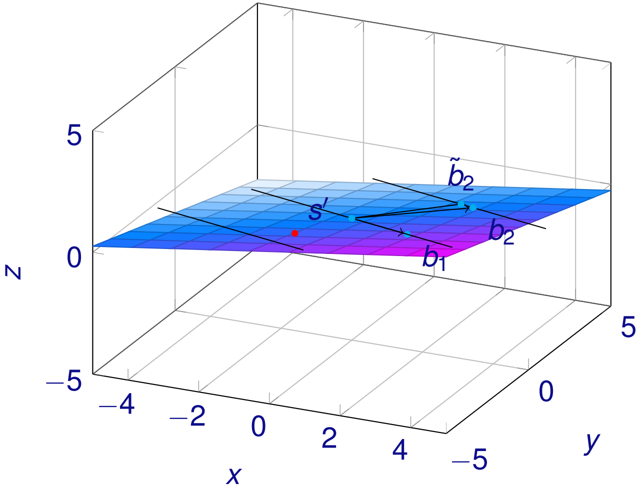
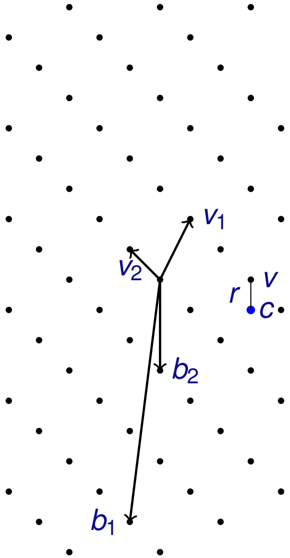


Part II - Crypto

Goldreich, Goldwasser and Halevi (GGH)



No security proof

Trapdoor: orthogonality

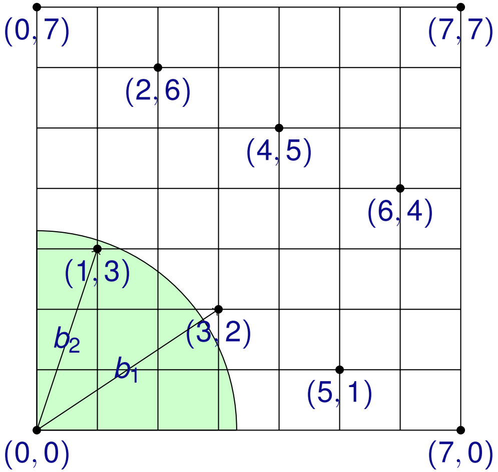
Good base: $V = (v_1, v_2)$

Bad base: $B = (b_1, b_2)$

Encrypt r: $c = v + r \pmod{B}$

Decrypt: $r = c - v$

Ajtai's Construction



$f_A(x) = Ax$

surjective

small x

(SIS problem)

collision: x, x'

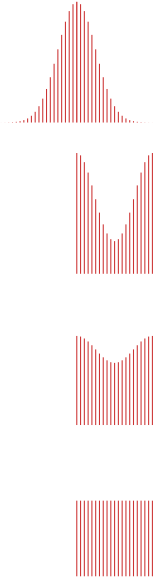
short vector: $(x - x')$

in $\Lambda_q^\perp$

worst to average

quantum reduction

Learning With Errors



Search problem:

Given $b_i = \langle a_i, s \rangle + e_i$

Find s

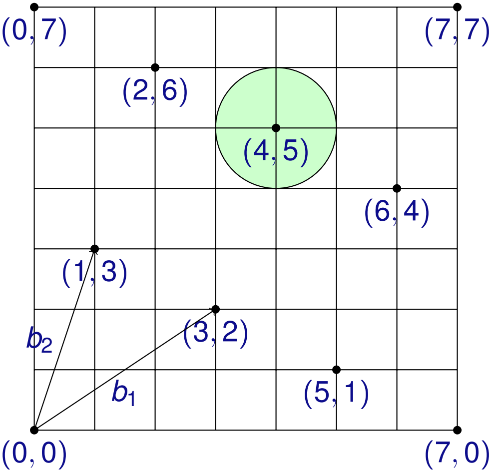
Decision problem:

Distinguish (a_i, b_i)

from uniform

Search to decision reduction

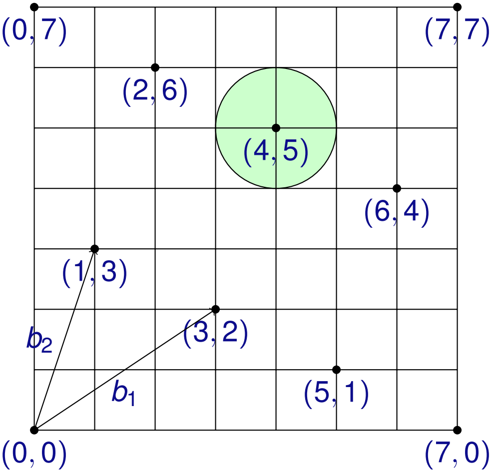
Learning With Errors



$$g_A(x) = Ax + e$$

injective

Learning With Errors



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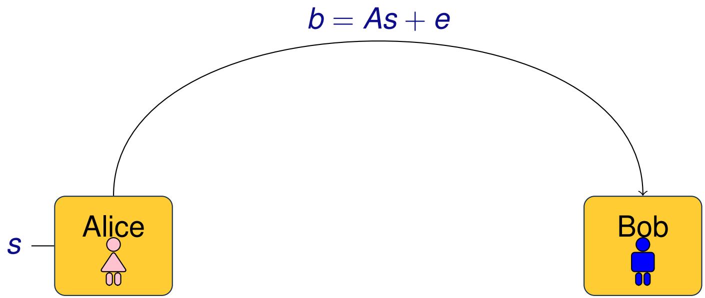
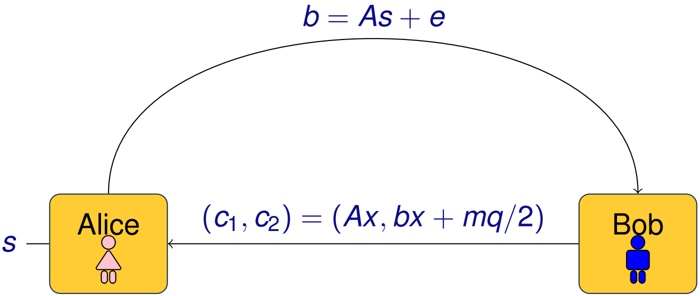
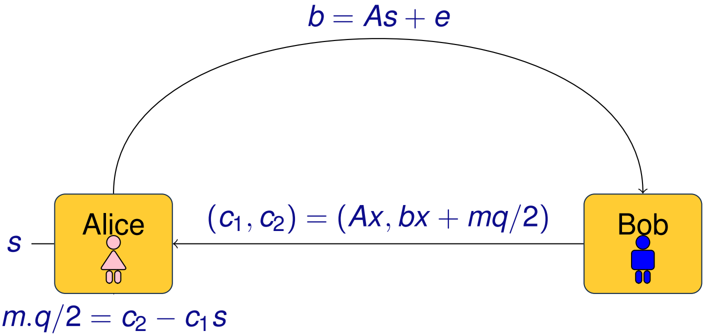
LWE Based Cryptosystem



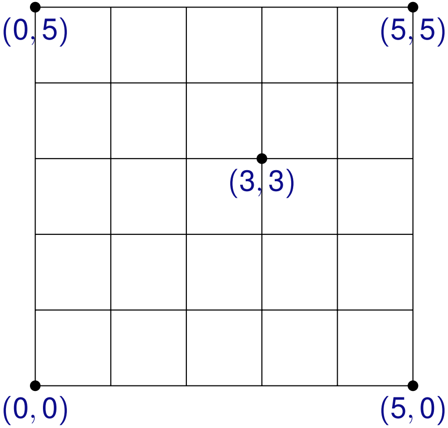
LWE Based Cryptosystem



LWE Based Cryptosystem



Cyclotomic Rings



$$\Phi_{2^n}(x) = (x^{2^{n-1}} + 1)$$

if $\zeta_{2^n} \in \mathbb{Z}_q$ then

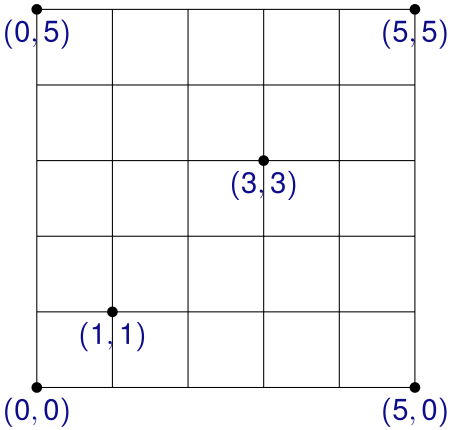
$$\Phi_{2^n} \equiv \prod_{i \in \mathbb{Z}_{2^n}^*} (x - \zeta_{2^n}^i)$$

Ring: $\mathbb{Z}_5[x]/(x^2 + 1)$

$$x^2 + 1 \equiv (x + 2)(x + 3)$$

$$a(x) = 3x + 3$$

Coefficient Representation

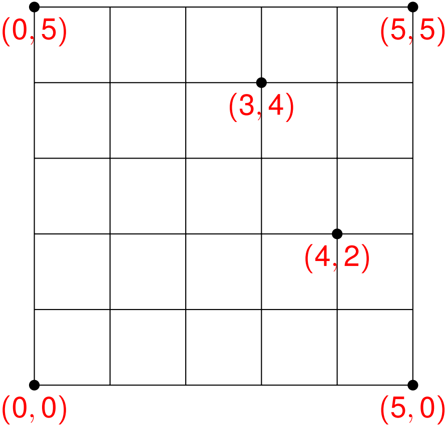


Ring: $\mathbb{Z}_5[x]/(x^2+1)$

$$x^2 + 1 \equiv (x+2)(x+3)$$
$$\equiv (x-3)(x-2)$$

$$a(x) = 3x + 3$$
$$2(3x+3) \equiv x + 1$$
$$\begin{bmatrix} 3 & 3 \end{bmatrix}^T, \begin{bmatrix} 1 & 1 \end{bmatrix}^T$$

Evaluation Representation



Ring: $\mathbb{Z}_5[x]/(x^2+1)$

$$x^2+1 \equiv (x+2)(x+3)$$

$$a(x) = 3x+3$$

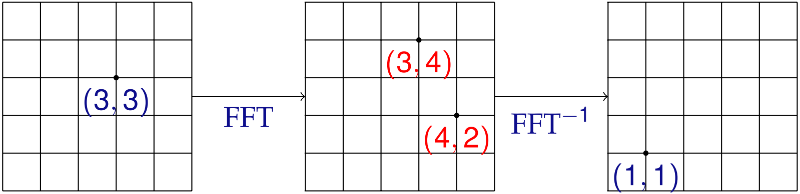
$$2a(x) \equiv x+1$$

$$a(2) \equiv 4, a(3) \equiv 2$$

$$\begin{bmatrix} 4 & 2 \end{bmatrix}^T, \begin{bmatrix} 3 & 4 \end{bmatrix}^T$$

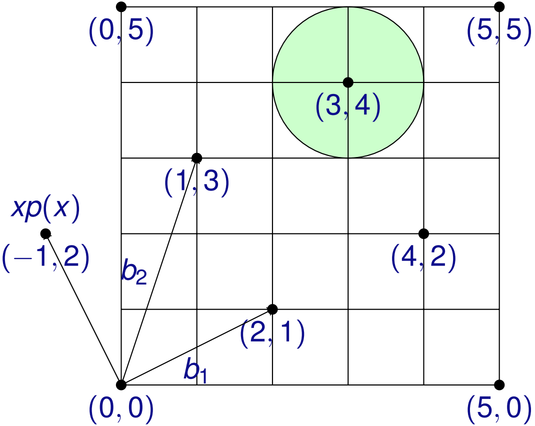
$\boxed{\text{FFT}}$

Cyclotomic Rings



$$\underbrace{\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}}_{\text{Vandermond}} \begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} \qquad \underbrace{\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}}_{\text{Vandermond inverse}} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Ring LWE



$$g_A(x) = Ax + e$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

ideal: $p(x) = x + 2$

Ring LWE

- Better reductions, better parameters

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  - Encryption, decryption, keygen: $\tilde{O}(n)$

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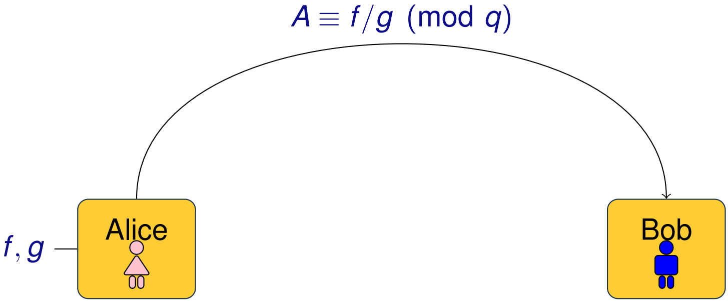
- Preimage sampleable trapdoors
  - Digital Signatures
- Cryptomania: IBE, ABE, FE, FHE

NTRU-like Cryptosystem [13]



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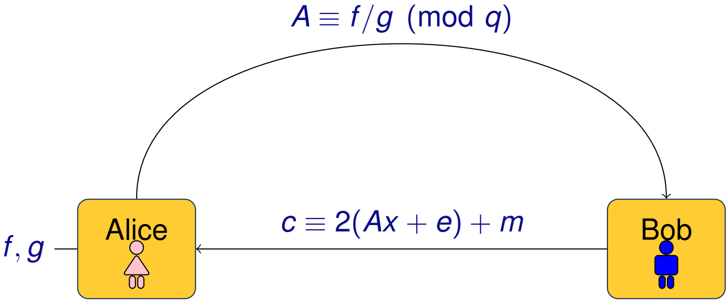
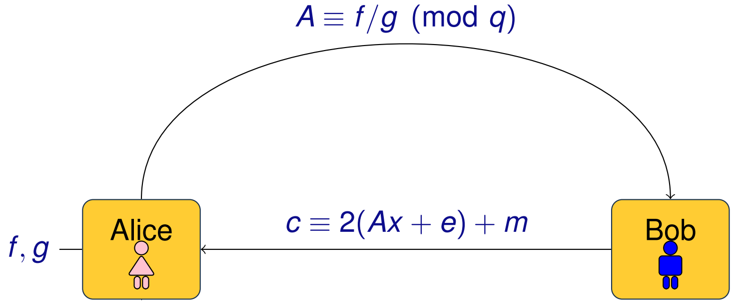


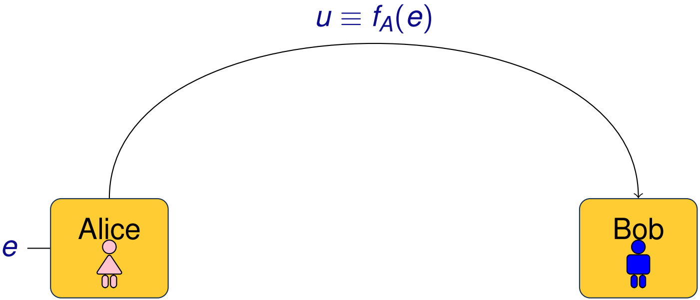
NTRU-like Cryptosystem [13]



$$cg \equiv 2(fx + eg) + mg$$

$$m \equiv (cg \pmod{2})/g$$

Dual LWE



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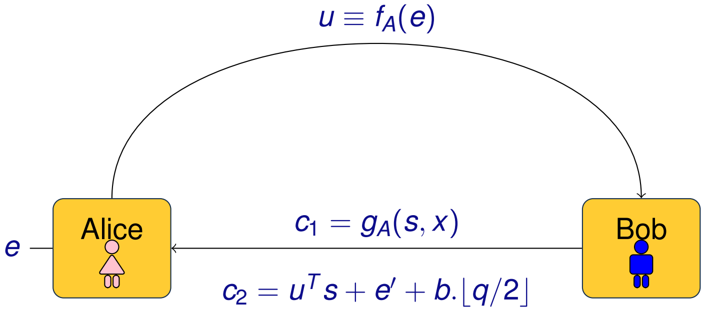

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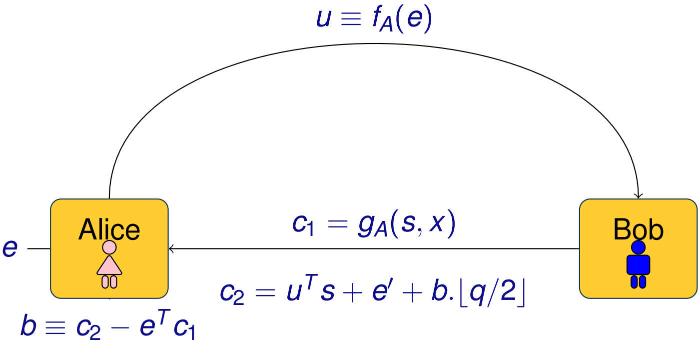
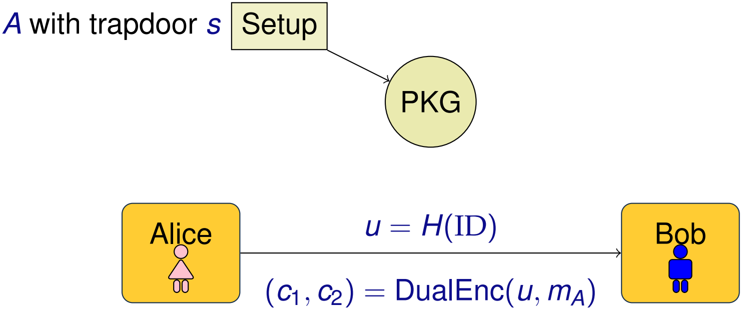

$u \equiv f_A(e)$
Alice
Bob
e
$c_1 = g_A(s, x)$
$c_2 = u^T s + e' + b.\lfloor q/2 \rfloor$
$b \equiv c_2 - e^T c_1$

Identity Based Encryption



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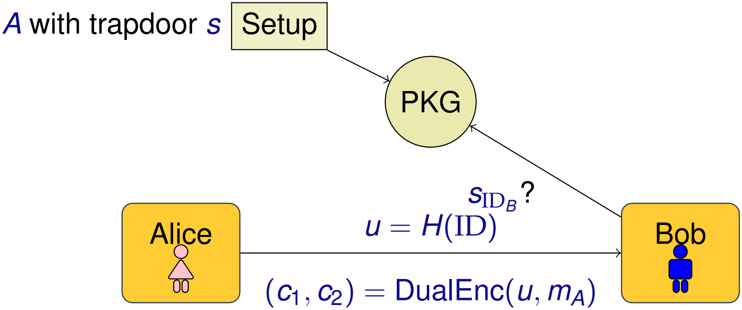

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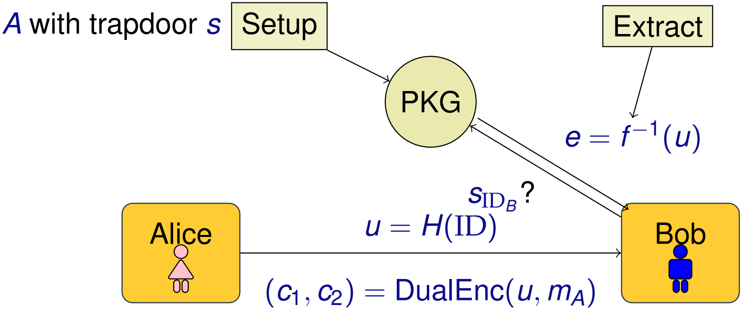

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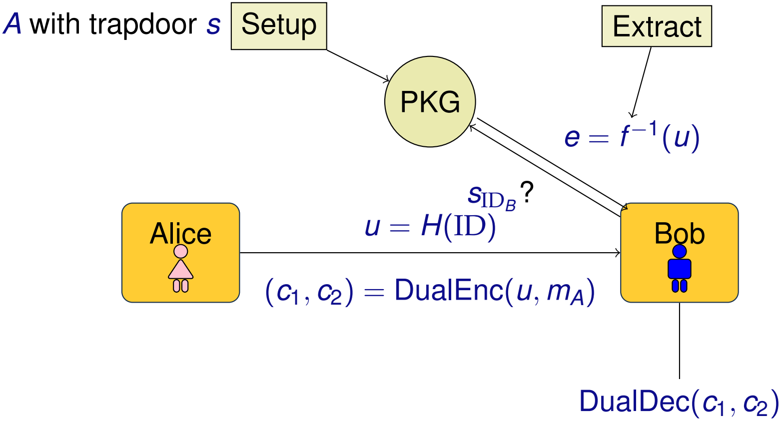

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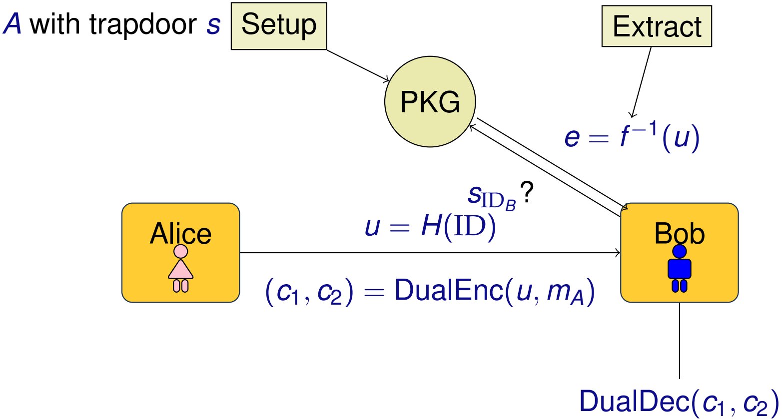
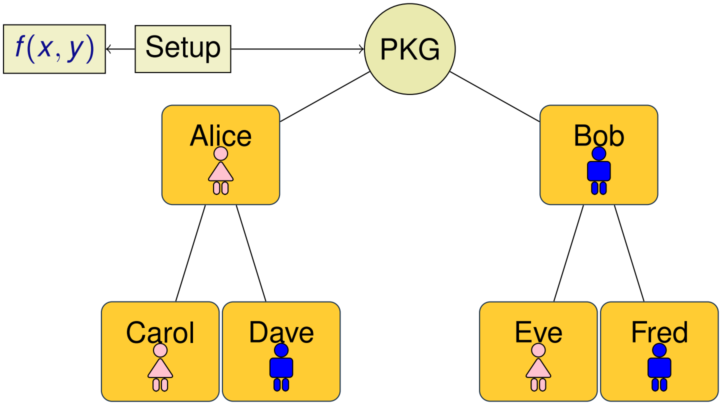


Functional Encryption



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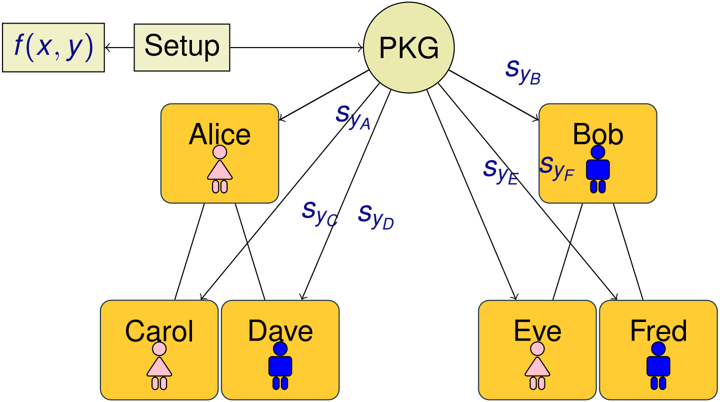

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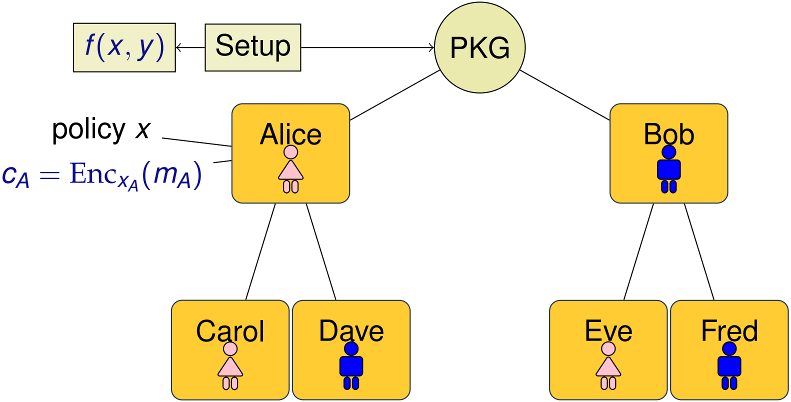

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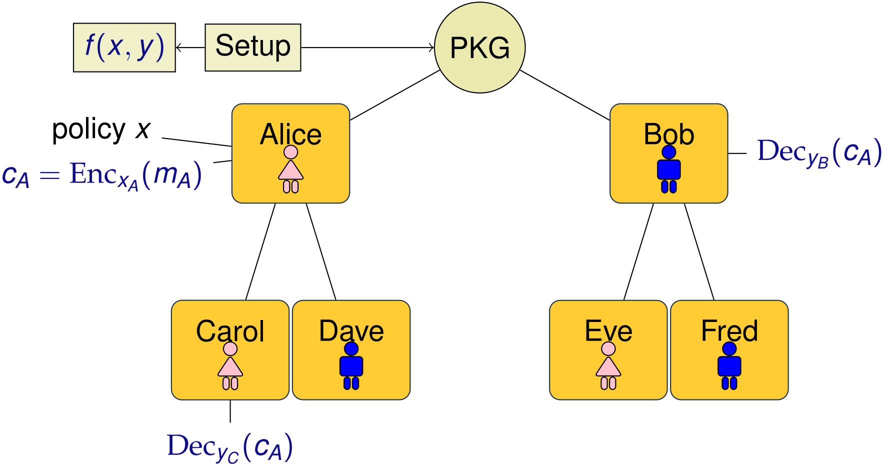
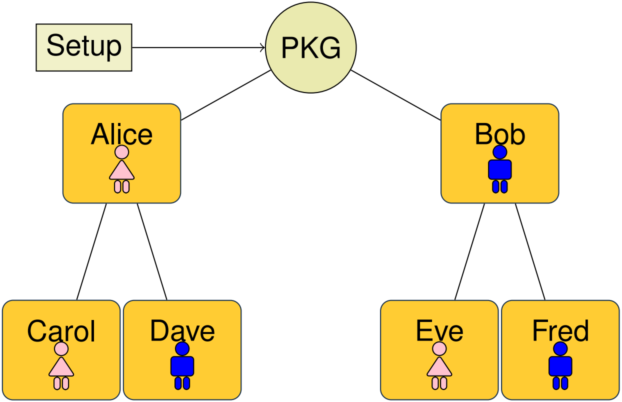


Attribute Based Encryption



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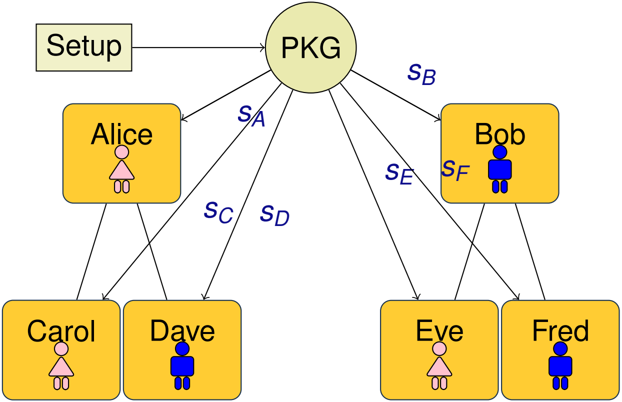

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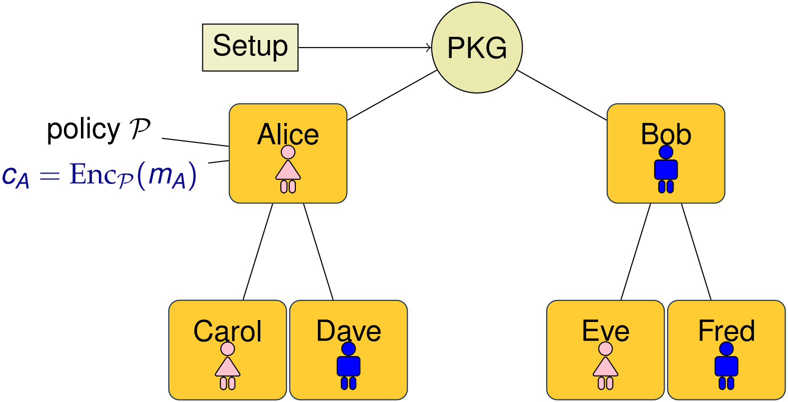

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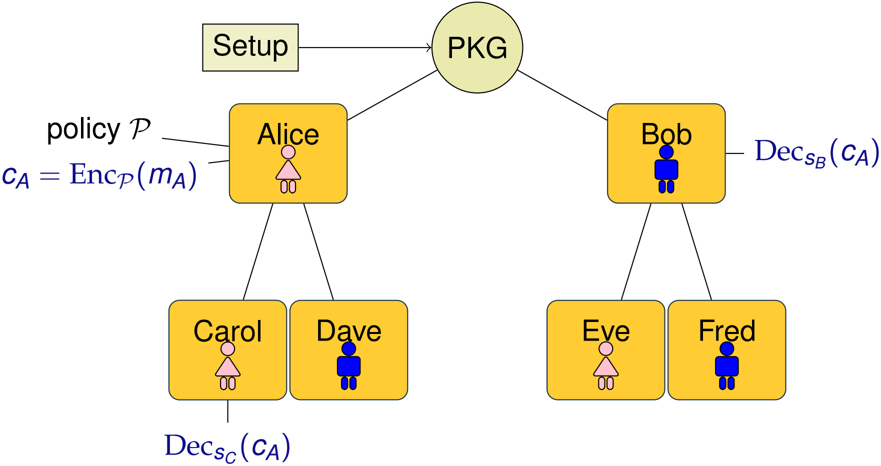

Fully Homomorphic Encryption

- Operations over encrypted messages

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- More with Zvika Brakerski

Conclusion

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- Lattices are not yet recommended by NSA!

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Thank you

Questions?