Feature Selection for Classification

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Group 10

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Slides: **http://www.comp.nus.edu.sg/~wongszec/group10.ppt**

Feature Selection for Classification

Agenda:

- Overview and general introduction. (pk)
- Four main steps in any feature selection methods. (pk)
- Categorization of the various methods. (pk)
- Algorithm = Relief, Branch & Bound. (pk)
- Algorithm = DTM, MDLM, POE+ACC, Focus. (sc)
- Algorithm = LVF, wrapper approach. (stan)
- Summary of the various method. (stan)
- Empirical comparison using some artificial data set. (stan)
- Guidelines in selecting the “right” method. (pk)

Feature Selection for Classification

(1) Overview.

- various feature selection methods since the 1970’s.
- common steps in all feature selection tasks.
- key concepts in feature selection algorithm.
- categorize 32 selection algorithms.
- run through some of the main algorithms.
- pros and cons of each algorithms.
- compare the performance of different methods.
- guideline to select the appropriate method.

Feature Selection for Classification

(2) What is a feature?

TRS_DT	TRS_TYP_CD	REF_DT	REF_NUM	CO_CD	GDS_CD	QTY	UT_CD	UT_PRIC
21/05/93	00001	04/05/93	25119	10002J	001M	10	CTN	22.000
21/05/93	00001	05/05/93	25124	10002J	032J	200	DOZ	1.370
21/05/93	00001	05/05/93	25124	10002J	033Q	500	DOZ	1.000
21/05/93	00001	13/05/93	25217	10002J	024K	5	CTN	21.000
21/05/93	00001	13/05/93	25216	10026H	006C	20	CTN	69.000
21/05/93	00001	13/05/93	25216	10026H	008Q	10	CTN	114.000
21/05/93	00001	14/05/93	25232	10026H	006C	10	CTN	69.000
21/05/93	00001	14/05/93	25235	10027E	003A	5	CTN	24.000
21/05/93	00001	14/05/93	25235	10027E	001M	5	CTN	24.000
21/05/93	00001	22/04/93	24974	10035E	009F	50	CTN	118.000
21/05/93	00001	27/04/93	25033	10035E	015A	375	GRS	72.000
21/05/93	00001	20/05/93	25313	10041Q	010F	10	CTN	26.000
21/05/93	00001	12/05/93	25197	10054R	002E	25	CTN	24.000

Feature Selection for Classification

(3)What is classification?

- main data mining task besides association-rule discovery.
- predictive nature - with a given set of features, predict the value of another feature.
- common scenario :

  - Given a large legacy data set.
  - Given a number of known classes.
  - Select an appropriate smaller training data set.
  - Build a model (eg. Decision tree).
  - Use the model to classify the actual data set into the defined classes.

Feature Selection for Classification

(4) Main focus of the author.

- survey various known feature selection methods
- to select subset of relevant feature
- to achieve classification accuracy.

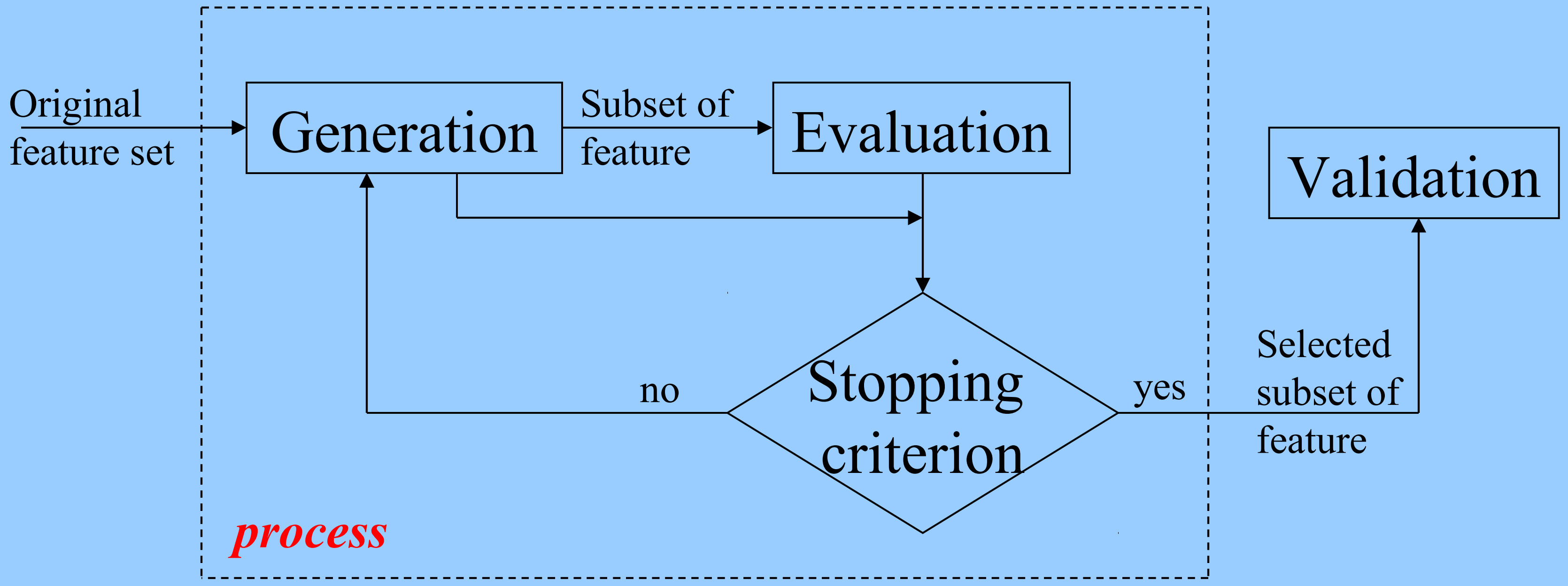
  Thus: relevancy -> correct prediction

(5) Why can't we use the full original feature set?

- too computational expensive to examine all features.
- not necessary to include all features
  (ie. irrelevant - gain no further information).

Feature Selection for Classification

(6) Four main steps in a feature selection method.



Generation = select feature subset candidate.
Evaluation = compute relevancy value of the subset.
Stopping criterion = determine whether subset is relevant.
Validation = verify subset validity.

Feature Selection for Classification

(7) Generation

- select candidate subset of feature for evaluation.
- Start = no feature, all feature, random feature subset.
- Subsequent = add, remove, add/remove.
- categorise feature selection = ways to generate feature subset candidate.
- 3 ways in how the feature space is examined.

(7.1) Complete

(7.2) Heuristic

(7.3) Random.

Feature Selection for Classification

(7.1) Complete/exhaustive

- examine all combinations of feature subset.
  {f1,f2,f3} => { {f1},{f2},{f3},{f1,f2},{f1,f3},{f2,f3},{f1,f2,f3} }
- order of the search space $O(2^p)$, p - # feature.
- optimal subset is achievable.
- too expensive if feature space is large.

(7.2) Heuristic

- selection is directed under certain guideline
  - selected feature taken out, no combination of feature.
  - candidate = { {f1,f2,f3}, {f2,f3}, {f3} }
- incremental generation of subsets.
- search space is smaller and faster in producing result.
- miss out features of high order relations (parity problem).
  - Some relevant feature subset may be omitted {f1,f2}.

Feature Selection for Classification

(7.3) Random

- no predefined way to select feature candidate.
- pick feature at random (ie. probabilistic approach).
- optimal subset depend on the number of try
  - which then rely on the available resource.
- require more user-defined input parameters.
  - result optimality will depend on how these parameters are defined.
  - eg. number of try

Feature Selection for Classification

(8) Evaluation

- determine the relevancy of the generated feature subset candidate towards the classification task.

  Rvalue = J(candidate subset)

  if (Rvalue > best_value) best_value = Rvalue

- 5 main type of evaluation functions.

  (8.1) distance (euclidean distance measure).

  (8.2) information (entropy, information gain, etc.)

  (8.3) dependency (correlation coefficient).

  (8.4) consistency (min-features bias).

  filter

  (8.5) classifier error rate (the classifier themselves).

  wrapper

Feature Selection for Classification

(8.1) Distance measure

- $z^2 = x^2 + y^2$
- select those features that
  support instances of the same class to
  stay within the same proximity.
- instances of same class should be closer in terms of distance
  than those from different class.

(8.2) Information measure

- entropy - measurement of information content.
- information gain of a feature : (eg. Induction of decision tree)
  gain(A) = I(p,n) - E(A)
  gain(A) = before A is branched - sum of all nodes after branched
- select A if gain(A) > gain(B).

Feature Selection for Classification

(8.3) Dependency measure

- correlation between a feature and a class label.
- how close is the feature related to the outcome of the class label?
- dependence between features = degree of redundancy.
  - if a feature is heavily dependence on another, than it is redundant.
- to determine correlation, we need some physical value.
  value = distance, information

Feature Selection for Classification

(8.4) Consistency measure

- two instances are *inconsistent*
  if they have *matching feature values*
  but group under *different class label*.

	f_1	f_2	class
instance 1	a	b	c1
instance 2	a	b	c2

inconsistent

- select {f1,f2}
  if in the training data set there exist no instances as above.
- heavily rely on the training data set.
- min-feature = want smallest subset with consistency.
- problem = 1 feature alone guarantee no inconsistency (eg. IC #).

Feature Selection for Classification

<u>Filter approach</u>

Original
feature set

Feature selection
- evaluation fn

selected
feature subset

classifier

- evaluation fn <> classifier
- ignored effect of selected subset on the performance of classifier.

<u>Wrapper approach</u>

Original
feature set

Feature selection
- classifier

selected
feature subset

classifier

- evaluation fn = classifier
- take classifier into account.
- loss generality.
- high degree of accuracy.

Feature Selection for Classification

(8.5) Classifier error rate.

- wrapper approach.
  error_rate = classifier(feature subset candidate)
  if (error_rate < predefined threshold) select the feature subset
- feature selection loss its generality, but gain accuracy towards the classification task.
- computationally very costly.

Feature Selection for Classification

(9) Comparison among the various evaluation method.

method	generality	time	accuracy
distance	yes	low	-
information	yes	low	-
dependency	yes	low	-
consistency	yes	moderate	-
classifier error rate	no	high	very high

generality = how general is the method towards diff. classifier?

time = how complex in terms of time?

accuracy = how accurate is the resulting classification task?

Feature Selection for Classification

(10) Author's categorization of feature selection methods.

Measures	Generation		
	Heuristic	***Complete***	***Random***
Distance	Relief	Branch & Bound (BB)	
Information	Decision Tree Method (DTM)	Minimal Description Length Method (MDLM)	
Dependency	Probability of Err & Ave Correlation Coefficient Method (POE+ACC)		
Consistency		Focus	LVF
Classifier Error Rate	SBS, SFS	AMB & B	LVW

Feature Selection for Classification

<u>(11.1) Relief</u> [generation=heuristic, evaluation=distance].

- Basic algorithm construct :

  - each feature is assigned cumulative weightage computed over a predefined number of sample data set selected from the training data set.

  - feature with weightage over a certain threshold is the selected feature subset.

- Assignment of weightage :

  - instances belongs to similar class should stay closer together than those in a different class.

  - near-hit instance = similar class.

  - near-miss instance = different class.

  - $W = W - \text{diff}(X,\text{nearhit})^2 + \text{diff}(X,\text{nearmiss})^2$

Feature Selection for Classification

1. selected_subset = {}

2. init. all feature weightage = 0 (eg. for 2 features : w_1=0, w_2=0)

3. for i = 1 to no_of_sample

   get one instance X from the training data set D.

   get nearhit H = instance in D where dist(X,H) is closest & X.class=H.class

   get nearmiss M = instance in D where dist(X,M) is closest & X.class<>M.class

   update weightage for all features :

   - weightage = weightage -diff(x,h)^2 +diff(x,m)^2

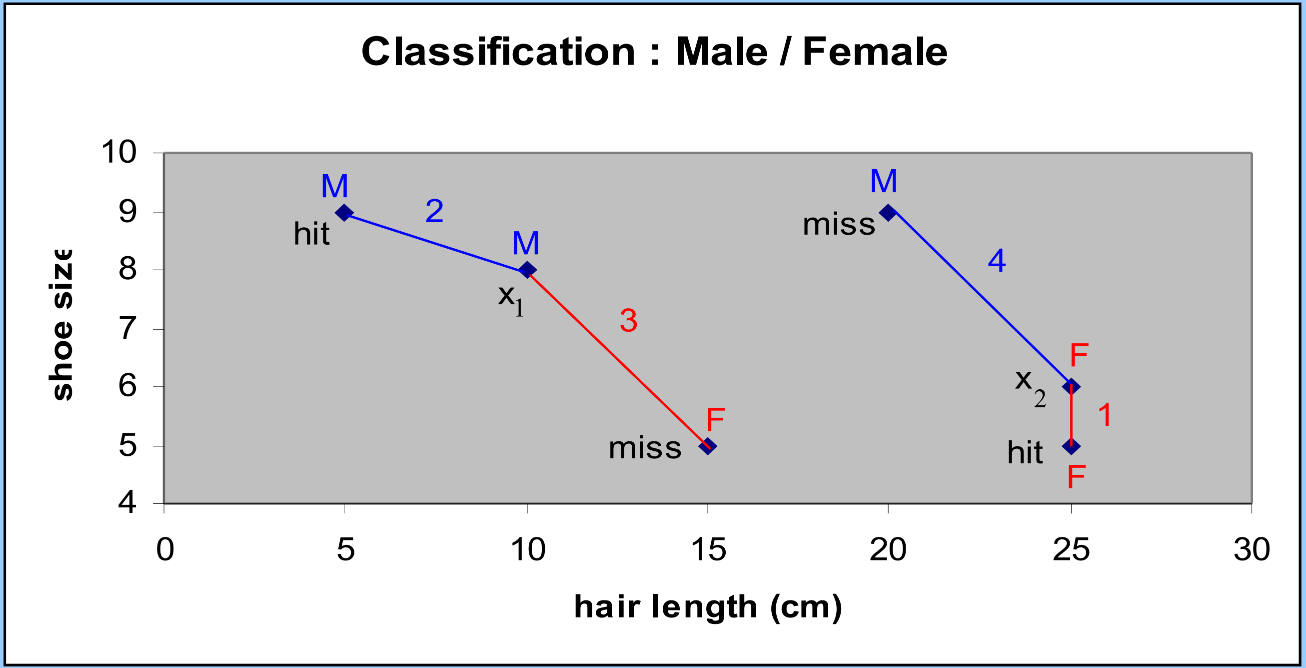
     eg. $\text{weightage}_1 = \text{weightage}_1 - \text{diff}(x_1,h_1)^2 + \text{diff}(x_1,m_1)^2$

     eg. $\text{weightage}_2 = \text{weightage}_2 - \text{diff}(x_2,h_2)^2 + \text{diff}(x_2,m_2)^2$

4. for j = 1 to no_of_feature (eg. 2)

   if weightage_j >= Threshold, add feature_j to selected_subset

Feature Selection for Classification



feature	x	w	$-(x\text{-hit})^2$	$+(x\text{-miss})^2$	=w	x	w	$-(x\text{-hit})^2$	$+(x\text{-miss})^2$	=w
shoe size	x_1	0	$-(4-5)^2$	$+(4-1)^2$	-1+9	x_2	8	$-(2-1)^2$	$+(2-5)^2$	+16
hair length	x_1	0	$-(2-1)^2$	$+(2-3)^2$	-1+1	x_2	0	$-(5-5)^2$	$+(5-4)^2$	+1

* if (threshold=5), the feature “shoe size” will be selected.

Feature Selection for Classification

- $W = W - diff(X,nearhit)^2 - diff(X,nearmiss)^2$

  - try to decrease weightage for instances belong to the same class (*note: their dist. diff. should be small).

  - try to increase weightage for instances belong to diff class (*note: their dist. diff. should be large).

  - If (W<=0), then sign of irrelevancy or redundancy.

  - If (W>0), then instances in diff. class is further apart as expected.

- Disadvantages:

  - applicable only to binary class problem.

  - insufficient training instances fool relief.

  - if most features are relevant, relief select all (even if not necessary).

- Advantages:

  - noise-tolerant.

  - unaffected by feature interaction (weightage is cumulative & det. collectively).

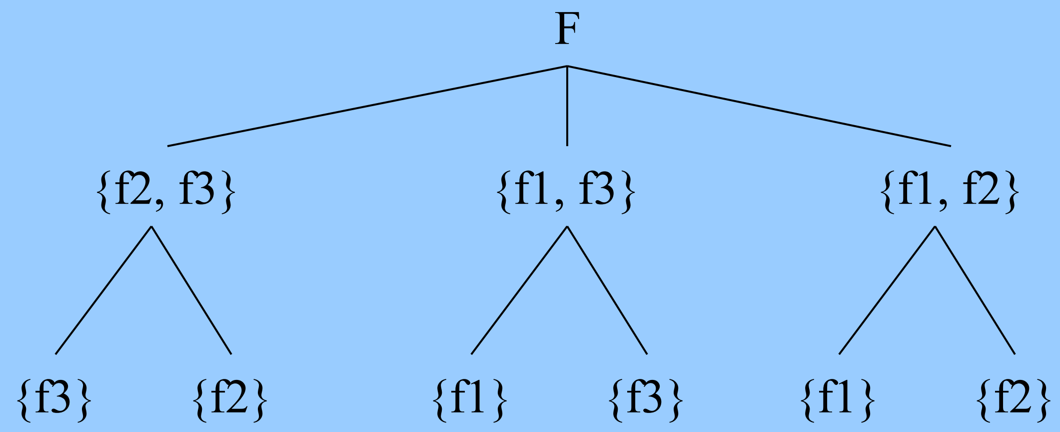
Feature Selection for Classification

(11.2) Branch & Bound. [generation=complete, evaluation=distance]

- is a very old method (1977).
- Modified assumption :
  - find a minimally size feature subset.
  - a bound/threshold is used to prune irrelevant branches.
- F(subset) < bound, remove from search tree (including all subsets).
- Model of feature set search tree.

Feature Selection for Classification

F = { f1, f2, f3 }



Category IV - Generation Heuristic / Evaluation Information

2 Methods:

- 1) Decision Tree Method (DTM)
  - Run C4.5 over training set.
  - The features that are selected are the union of all features in the pruned decision tree produced by C4.5.
  - An information based function selects the feature at each node of the decision tree

Category IV - Generation Heuristic / Evaluation Information

DTM Algorithm. Parameters (D)

1. $T = \varnothing$
2. Apply C4.5 to training set, D
3. Append all features appearing in the pruned decision tree to T
4. Return T

D = Training Set

Category IV - Generation Heuristic / Evaluation Information

C4.5

- Uses Information based Heuristic for node selection.
- $I(p,n) = -\left(\frac{p}{p+n}\right)\log_2\left(\frac{p}{p+n}\right) - \left(\frac{n}{p+n}\right)\log_2\left(\frac{n}{p+n}\right)$
  - p = # of instances of class label 1
  - n = # of instances of class label 0

- Entropy - "a measure of the loss of information in a transmitted signal or message".
- $E(F_i) = \left(\frac{p_0+n_0}{p+n}\right)I(p_0,n_0) + \left(\frac{p_1+n_1}{p+n}\right)I(p_1,n_1)$
  - p_x = # of instances with feature value = x, class value = 1 (positive)
  - n_x = # of instances with feature value = x, class value = 0 (negative)

- $E(C) = \frac{6+2}{16}I(6,2) + \frac{1+7}{16}I(1,7) = 0.677421$

Category IV - Generation Heuristic / Evaluation Information

- Feature to be selected as root of decision tree has minimum entropy.
- Root node partitions, based on the values of the selected feature, instances into two nodes.
- For each of the two sub-nodes, apply the formula to compute entropy for remaining features. Select the one with minimum entropy as node feature.
- Stop when each partition contains instances of a single class or until the test offers no further improvement.
- C4.5 returns a pruned-tree that avoids over-fitting.

$\therefore$ The union of all features in the pruned decision tree is returned as *T*.

Category IV - Generation Heuristic / Evaluation Information

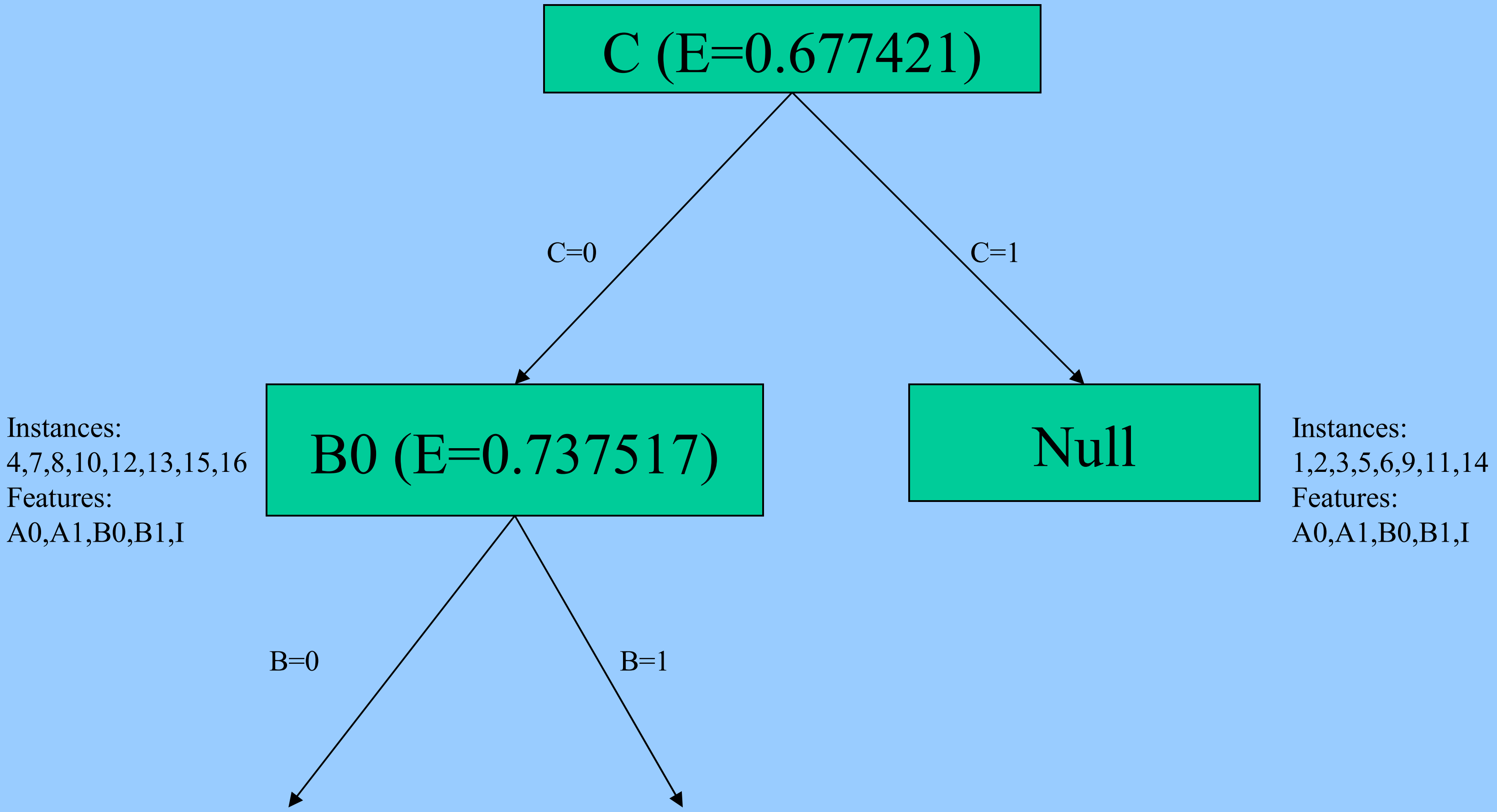
Hand-run of CorrAL Dataset:

- Computation of Entropy across all features for selecting root of the decision tree :

Feature - *F*	**E(*F*)**
I	0.850603
B1	0.882856
B0	0.882856
A1	0.882856
A0	0.882856
C	0.677421

∀ ∴ C is selected as the root because it has the lowest entropy.

Category IV - Generation Heuristic / Evaluation Information



DTM returns $\{ A_0, A_1, B_0, B_1, C \}$

Category IV - Generation Heuristic / Evaluation Information

2) Koller and Sahami's method

- Intuition:
  - Eliminate any feature that does not contribute any additional information to the rest of the features.
- Implementation attempts to approximate a Markov Blanket.
- However, it is suboptimal due to naïve approximations.

Category V - Generation Complete / Evaluation Information

1 Method:

- Minimum Description Length Method (MDLM)
  - Eliminate useless (irrelevant and/or redundant) features
  - 2 Subsets: U and V, $U \cap V = \emptyset$, $U \cup V = S$

    $\forall$ v, v $\in V$, if $\boldsymbol{F}$(u) = v, u $\in U$ where $\boldsymbol{F}$ is a fixed non-class dependent function, then features in V becomes useless when is U becomes known.
  - $\boldsymbol{F}$ is formulated as an expression that relates:
    - the # of bits required to transmit the classes of the instances
    - the optimal parameters
    - the useful features
    - the useless features
  - Task is to determine U and V.

Category V - Generation Complete / Evaluation Information

- Uses Minimum Description Length Criterion (MDLC)
  - MDL is a mathematical model for Occam's Razor.
  - Occam's Razor - principle of preferring simple models over complex models.
- MDLM searches all possible subsets: 2^N
- Outputs the subset satisfying MDLC
- MDLM finds useful features only if the observations (the instances) are Gaussian

Category V - Generation Complete / Evaluation Information

MDLM Algorithm. Parameters (*D*):

1. Set $MDL = \infty$
2. For all feature subsets *L*:

1.1 Compute $Length_L = \sum_{i=1}^{i=q} \frac{P_i}{2} \log \frac{|D_L(i)|}{|D_L|} + h_L$

where $h_L = \frac{1}{2}(N-M)(N+M+3)\log P + \sum_{i=1}^{i=q} M(M+3)\log P_i$,

N – total number of features,

M – number of features in the candidate subset,

P – total number of instances in D,

P_i – number of instances with class label i,

q – total number of class labels,

D_L – covariance matrix formed from all the useful feature vectors,

$D_L(i)$ – covariance matrix formed from the useful feature vectors of class i,

$|.|$ – denotes determinant.

$T = L$, $MDL = Length_L$

3. Return *T*   *D* = Training Set

Category V - Generation Complete / Evaluation Information

- Suggested implementation
  - For all feature subsets:
    - 1. Calculate the covariance matrices of the whole feature vectors for all classes: D_L
    - 2. Calculate the covariance matrices of the whole feature vectors for each separate class: D_L(i)
    - 3. Obtain the covariance matrix for useful subsets as sub-matrixes of D_L and D_L(i)
    - 4. Compute the determinants of the sub-matrices D_L and D_L(i)
    - 5. Compute $Length_{\mathrm{L}}$ given 1,2,3,4 as in step 2 of the algorithm
- Return subset that has the minimum description length.
- Hand-run of CorrAL dataset returns {C} with minimum description length of 119.582.

Category VII - Generation Heuristic/Evaluation Dependence

2 methods

- 1) POE + ACC (Probability of Error and Average Correlation Coefficient)
  - First feature selected is feature with smallest probability of error (P_e).
  - The next feature selected is feature that produces minimum weighted sum of P_e and average correlation coefficient ACC.
  - ACC is mean of correlation coefficients of all candidate features with features previously selected at that point.
  - This method can rank all the features based on the weighted sum.
  - Stopping criterion is the required number of features.
  - The required parameters are the number of features and the weights w_1 and w_2.

Category VII - Generation Heuristic/Evaluation Dependence

POE + ACC Algorithm .Parameters (M, w_1, w_2)

1. $T = \varnothing$
2. Find feature with minimum P_e and append to T
3. For i = 1 to M-1

   Find the next feature with minimum $w_1(P_e) + w_2(ACC)$

   Append it to T
4. Return T

M = Required number of features

w_1= Weight for POE

w_2= Weight for ACC

Category VII - Generation Heuristic/Evaluation Dependence

- To calculate P_e
  - First compute the a priori probability of different classes
  - For each feature, calculate the class-conditional probabilities given the class label.
  - Then for each feature value, find the class label for which the product of a priori class probability and class-conditional probability given the class label is a maximum
  - Finally count the number of mismatches between the actual and predicted class values and select the feature with minimum mismatches
- To calculate ACC:
  - Compute correlation coefficient of the candidate feature *x*, with each feature previous selected. (Correlation coefficient measures the amount of linear association between any 2 random variables) :

$$\mathrm{ACC}(x) = (\Sigma^{n}\ \mathrm{Corr}(x,y)) / n \quad \text{where } n = |T|,\ y \in T$$

Category VII - Generation Heuristic/Evaluation Dependence

Hand-run of CorrAL Dataset:

- A priori class probabilities of *D:*
  - for class 0 = 9/16 , class 1 = 7/16
- For feature *C*: class-conditional probability calculation:

	class = 0	*class* = 1
P (*C*=0)	2/9	6/7
P (*C*=1)	7/9	1/7

- Calculating product of a priori class probability and class-conditional probability given the class label:

	x = 0	*x* = 1
P (*C*=0 | Class = *x*)	2/9 * 9/16 = 0.125	6/7 * 7/16 = 0.375
P (*C*=1 | Class = *x*)	7/9 * 9/16 = 0.4375	1/7 * 7/16 = 0.0625

- Thus when *C* takes value of 0, the prediction is class = 1 and when *C* takes the value of 1, the prediction is class = 0.

Category VII - Generation Heuristic/Evaluation Dependence

- Using this, the number of mismatches between the actual and predicted class values is counted to be 3 (instances 7, 10 and 14)

∀ ∴ P_e of feature C = 3/16 or 0.1875.

- According to the author, this is the minimum among all the features and is selected as the first feature.
- In the second step, the P_e and ACC (of all remaining features $\{A_0, A_1, B_0, B_1, I\}$ with feature C) are calculated to choose the feature with minimum $[w_1(P_e) + w_2(ACC)]$
- Stop when required number of features have been selected.
- For hand-run of CorrAL, subset $\{ C, A_0, B_0, I \}$ is selected.

Category VII - Generation Heuristic/Evaluation Dependence

- 2) PRESET
  - Uses the concept of a rough set
  - First find a reduct and remove all features not appearing in the reduct (a reduct of a set P classifies instances equally well as P does)
  - Then rank features based on their significance measure (which is based on dependency of attributes)

Category XI - Generation Complete/Evaluation Consistency

3 Methods:

- 1) Focus
  - Implements the Min-Features bias
  - Prefers consistent hypotheses definable over as few features as possible
  - Unable to handle noise but may be modified to allow a certain percentage of inconsistency

Category XI - Generation Complete/Evaluation Consistency

Focus Algorithm. Parameters (D, S)

1. $T = S$
2. For i = 0 to $N-1$

    For each subset L of size i

        If no inconsistency in the training set D then

            $T = L$

            return T

D = Training Set

S = Original Feature Set

Category XI - Generation Complete/Evaluation Consistency

- Focus performs breath-first generation of feature subsets:-
  - It first generates subsets of size one, then two, and so on.
  - For each subset generated, check whether there are any inconsistencies.
  - A subset is inconsistent when there are at least two instances in the dataset having equal values for all the features under examination. Eg, for subset $\{A_0\}$, instances 1 and 4 have the same A_0 instance value (ie:- 0) but different class labels (0 and 1 respectively)
  - Continues until it finds the first subset that is not inconsistent or when the search is complete.

Category XI - Generation Complete/Evaluation Consistency

Hand-run of CorrAL Dataset:

- Consistent feature sets are:
  - $\{ A_0, A_1, B_0, B_1 \}$
  - $\{ A_0, A_1, B_0, B_1, I \}$
  - $\{ A_0, A_1, B_0, B_1, C \}$
  - $\{ A_0, A_1, B_0, B_1, I, C \}$
- However Focus returns the smallest consistent subset that is $\{A_0, A_1, B_0, B_1\}$.
- Trivial implementation of Focus:
  - http://www.comp.nus.edu.sg/~wongszec/cs6203_focus.pl
  - To run, type: *perl cs6203_focus.pl*

Category XI - Generation Complete/Evaluation Consistency

- 2) Schlimmer's Method
  - Variant of Focus: Uses a systematic enumeration scheme as generation procedure and the inconsistent criterion as the evaluation function
  - Uses a heuristic function that makes the search for the optimal subset faster.
- 3) MIFES_1
  - Also very similar to Focus: Represents the set of instances in the form of a matrix.

CATEGORY XII (Consistency – Random)

LVF Algorithm

- Las Vegas Algorithm
- Randomly search the space of instances which makes probabilistic choices more faster to an optimal solution
- For each candidate subsets, **LVF** calculates an inconsistency count based on the intuition
- An inconsistency threshold is fixed in the beginning (Default = 0)
- Any subsets with inconsistency rate > threshold, **REJECT**

CATEGORY XII (Consistency – Random)

LVF Algorithm

- **INPUT** MAX-TRIES
  D - Dataset
  N - Number of attributes
  γ - Allowable inconsistency rate

- **OUTPUT** sets of M features satisfying
  the inconsistency rate

CATEGORY XII (Consistency – Random)

LVF Algorithm

$C_{best} = N$;

FOR I = 1 to MAX-TRIES

S = randomSet(seed);

C = numOfFeatures(S);

IF $(C < C_{best})$

IF (InconCheck(S,D) < γ);

$S_{best} = S$; $C_{best} = C$;

print_Current_Best(S)

ELSE IF (($C = C_{best}$) **AND** (InConCheck(S,D) < γ))

print_Current_Best(S)

END FOR

CATEGORY XII (Consistency – Random)

LVF Algorithm

ADVANTAGE

- Find optimal subset even for database with Noise
- User does not have to wait too long for a good subset
- Efficient and simple to implement, guarantee to find optimal subset if resources permit

DISADVANTAGE

- It take more time to find the optimal subset (whether the data-set is consistent or not)

FILTER VS WRAPPER

FILTER METHOD

Consider attributes independently from the induction algorithm

- Exploit general characteristics of the training set (statistics: regression tests)
- Filtering (of irrelevant attributes) occurs before the training

FILTER VS WRAPPER

WRAPPER METHOD

- Generate a set of candidate features
- Run the learning method with each of them
- Use the accuracy of the results for evaluation (either training set or a separate validation set)

WRAPPER METHOD

- Evaluation Criteria (**Classifier Error Rate**)
  - ≈ Features are selected using the classifier
  - ≈ Use these selected features in predicting the class labels of unseen instances
  - ≈ Accuracy is very high
- Use actual target classification algorithm to evaluate accuracy of each candidate subset
- Generation method: **heuristics**, **complete** or **random**
- The feature subset selection algorithm conducts a search for a good subset using the induction algorithm, as part of evaluation function

WRAPPER METHOD

DISADVANTAGE

- Wrapper very slow
- Higher Computation Cost
- Wrapper has danger of overfitting

CATEGORY XIII: CER – Heuristics

SFS (Sequential Forward Selection)

- Begins with zero attributes
- Evaluates all features subsets w/ exactly 1 feature
- Selects the one with the best performance
- Adds to this subsets the feature that yields the best performance for subsets of next larger size
- If **EVAL()** is a heuristics measure, the feature selection algorithm acts as a filter, extracting features to be used by the main algorithm; If it is the actual accuracy, it acts as a wrapper around that algorithm

CATEGORY XIII: CER – Heuristics

SFS (Sequential Forward Selection)

```
SS = 0
BestEval = 0
REPEAT
    BestF = None
    FOR each feature F in FS AND NOT in SS
      SS' = SS ∪ {F}
      IF Eval(SS') > BestEval THEN
                BestF = F; BestEval = Eval(SS')
    IF BestF <> None THEN SS = SS ∪ {BestF}
UNTIL BestF = None OR SS = FS
RETURN SS
```

CATEGORY XIII: CER – Heuristics

SBS (Sequential Backward Selection)

- Begins with all features
- Repeatedly removes a feature whose removal yields the maximal performance improvement

CATEGORY XIII: CER – Heuristics

SBS (Sequential Backward Selection)

SS =FS
BestEval = Eval(SS)
REPEAT
WorstF = None
FOR each feature in F in FS
SS' = SS - {F}
IF Eval(SS') >= BestEval **THEN**
WorstF = F; BestEval = Eval(SS')
IF WorstF <> None **THEN** SS = SS - {WorstF}
UNTIL WorstF = None **OR** SS = 0
RETURN SS

CATEGORY XIII: CER – Complete

ABB Algorithm

- Combat the disadvantage of **B&B** by permitting evaluation functions that are not monotonic.
- The bound is the inconsistency rate of dataset with the full set of features.

CATEGORY XIII: CER – Complete

ABB Algorithm

- Legitimate test: Determine whether a subset is a child note of a pruned node, by applying **Hamming distance**.
- **InConCal()** calculates the consistency rate of data given a feature subsets by ensuring :
  - No duplicate subset will be generated
  - No child of pruned node (**Hamming distance**)

CATEGORY XIII: CER – Complete

ABB Algorithm

□ = inConCal(S, D);

PROCEDURE ABB(S,D)

FOR all feature *f* in S

$S_1 = S - f$; enQueue(Q_1, S_1);

END FOR

WHILE notEmpty(Q)

S_2 = deQueue(Q);

IF (S_2 is legitimate $\wedge$ inConCal(S_2,D) <= ▪)

ABB(S_2, D);

END WHILE

END

CATEGORY XIII: CER – Complete

ABB Algorithm

- **ABB** expands the search space quickly but is inefficient in reducing the search space although it guarantee optimal results
- Simple to implement and guarantees optimal subsets of features
- **ABB** removes irrelevant, redundant, and/or correlated features even with the presence of noise
- Performance of a classifier with the features selected by **ABB** also improves

CATEGORY XIII: CER – Random

LVW Algorithm

- Las Vegas Algorithm
- Probabilistic choices of subsets
- Find Optimal Solution, if given sufficient long time
- Apply Induction algorithm to obtain estimated error rate
- It uses randomness to guide their search, in such a way that a correct solution is guaranteed even if unfortunate choices are made

CATEGORY XIII: CER – Random

LVW Algorithm

```
Err = 0; k = 0; C = 100;
REPEAT
    S1 = randomSet(); C1 = numOfFeatures(S1);
    err1 = LearnAlgo(S1, D_train, NULL);
    IF (err1 < err) OR (err1 = err AND C1 < C))
    output the current best;
    k = 0; err = err1; C = C1; S = S1;
    END IF
    k = k + 1;
UNTIL err is not updated for K times;
err2 = LearnAlgo(S, D_train, D_test);
```

CATEGORY XIII: CER – Random

LVW Algorithm

- **LVW** can reduce the number of features and improve the accuracy
- Not recommended in applications where time is critical factor
- Slowness is caused by learning algorithm

EMPIRICAL COMPARISON

- Test Datasets
  - ≈ Artificial
  - ≈ Consists of **Relevant** and **Irrelevant** Features
  - ≈ Know beforehand which features are relevant and which are not
- Procedure
  - ≈ Compare Generated subset with the known relevant features

CHARACTERISTIC OF TEST DATASETS

	CORRAL	PAR3+3	MONK3
Relevant	4	3	3
Irrelevant	1	3	3
Correlated	1	0	0
Redundant	0	3	0
Noisy	NO	NO	YES

RESULTS

- Different methods works well under different conditions
  - ≈ **RELIEF** can handle noise, but not redundant or correlated features
  - ≈ **FOCUS** can detect redundant features, but not when data is noisy
- No single method works under all conditions
- Finding a good feature subset is an important problem for real datasets. A good subset can
  - ≈ Simplify data description
  - ≈ Reduce the task of data collection
  - ≈ Improve accuracy and performance

RESULTS

- Handle Discrete? Continuos? Nominal?
- Multiple Class size?
- Large Data size?
- Handle Noise?
- If data is not noisy, able to produce optimal subset?

Feature Selection for Classification

Some Guidelines in picking the "right" method?

Based on the following 5 areas.
(i.e. mainly related to the characteristic of data set on hand).

- Data types - continuous, discrete, nominal
- Data size - large data set?
- Classes - ability to handle multiple classes (non binary)?
- Noise - ability to handle noisy data?
- Optimal subset - produce optimal subset if data not noisy?

Feature Selection for Classification

Comparison table of the discussed method.

Method	Generation	Evaluation	Contin.	Discrete	Nominal	Large Dataset	Multiple Classes	Handle Noise	Optimal Subset
B & B	complete	distance	y	y	n	-	y	-	y++
MDLM	complete	information	y	y	n	-	y	-	n
Focus	complete	consistency	n	y	y	n	y	n	y
Relief	heuristic	distance	y	y	y	y	n	y	n
DTM	heuristic	information	y	y	y	y	y	-	n
POE+ACC	heuristic	dependency	y	y	y	-	y	-	n
LVF	random	consistency	n	y	y	y	y	y*	y**

-	method does not discuss about the particular characteristic.
y++	if certain assumptions are valid.
y*	user is required to provide the noise level.
y**	provided there are enough resources.

*note : "classifier error rate" not included (ie. Depend on specify classifier).