

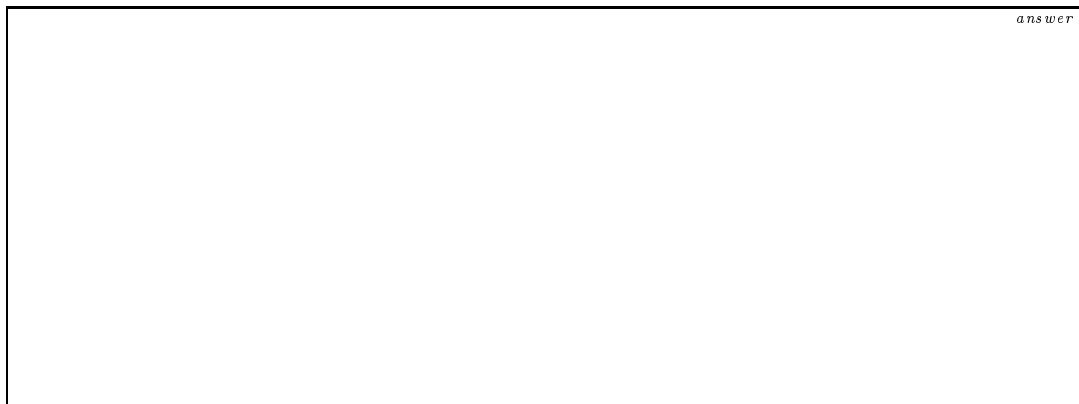


- (a) Write formulas for the counting variables  $f_I$ ,  $f_S$ ,  $g_I$ ,  $g_S$ , and  $h_S$ , as a function of  $n$ , for If you cannot find exact formulas, or the value depends on the list  $x$ , find the best lower and upper bounds that you can.



*answer*

- (b) What can you say about the relative efficiency of the two algorithms?

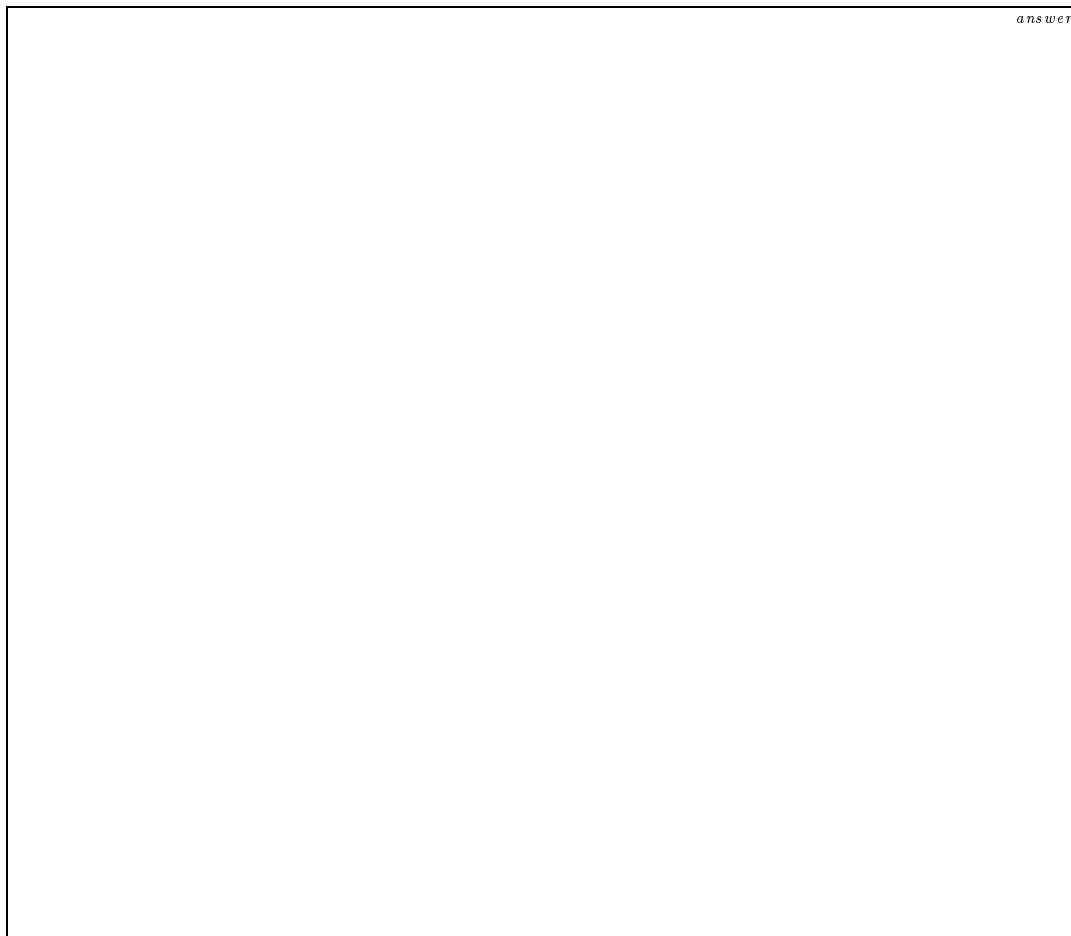


*answer*

2. This question refers to algorithm  $H$  below, that takes as inputs a natural number  $n$ , and a list  $X$  of  $n$  integers, and returns an integer  $m$ . (The algorithm does not solve any interesting problem, it is just an exercise in analysis.)

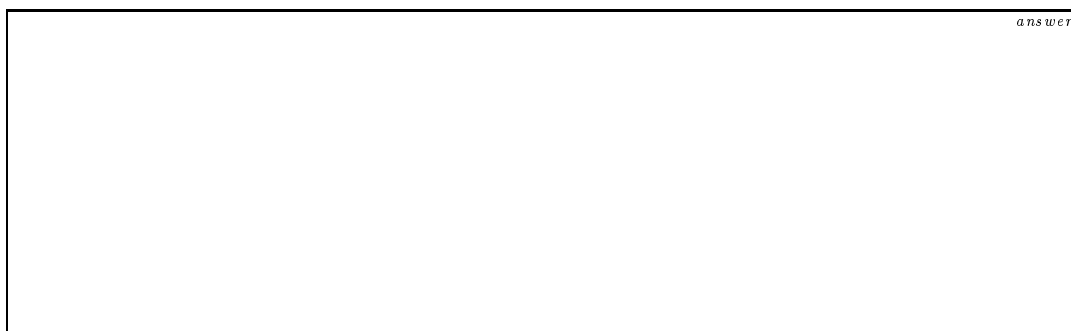
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Algorithm  $H(n, X)$ 
[f1]    $m \leftarrow 0; i \leftarrow 0;$ 
        while
[f2]    $i < n$ 
        do
[f3]    $j \leftarrow i;$ 
        while
[f4]    $j < n$ 
        do
[f5]    $A \leftarrow 0;$ 
[f6]    $r \leftarrow i;$ 
        while
[f7]    $r \leq j$ 
        do
[f8]    $A \leftarrow 2A + X_r;$ 
[f9]    $r \leftarrow r + 1;$ 
        endwhile;
[f10]  if  $A = 0$  then
[f11]   $m \leftarrow m + 1;$ 
        endif;
[f12]   $j \leftarrow j + 1;$ 
        endwhile;
[f13]   $i \leftarrow i + 1;$ 
        endwhile;
[f14]  return  $m;$ 
```

- (a) Write the Kirchoff laws that relate the counting variables  $f_1, f_2, \dots$ . (Drawing a flowchart of the algorithm may help.)



*answer*

- (b) Identify a “key” subset of the counting variables  $f_1, f_2, \dots$ , such that any other counting variable can be computed from that set, using the Kirchoff laws.



*answer*

- (c) Give formulas, as a function of  $n$ , for those key variables. If some variable depends on the matrix  $X$ , not only on  $n$ , give the best upper and lower bounds for that variable that you can find.

*answer*