Volumetric Image Visualization

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Volumetric Image Acquisition

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- Confocal microscopy measures the laser light reflection at a number of focus planes across the material.
- T1-weighted magnetic resonance (MR-T1) measures the longitudinal relaxation time of spins in hydrogen nuclei of the material, after turning on and off an external magnetic field.
- Computerized tomography (CT) measures the X-ray attenuation through the material on a number of projection planes.

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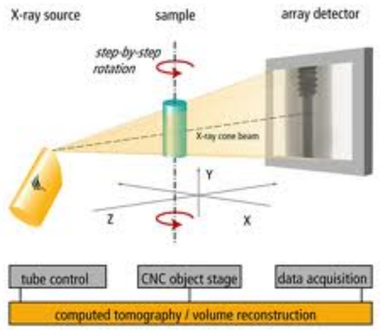

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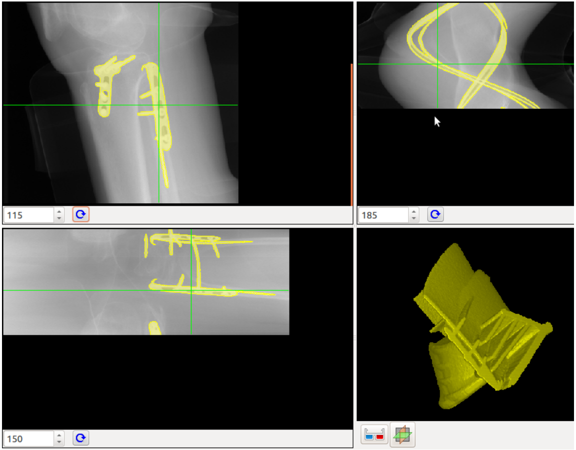

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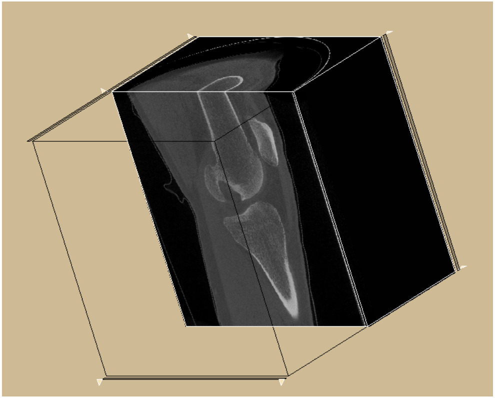

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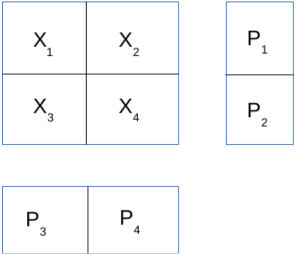
The 3D image reconstruction is an inverse problem.



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$A X = P$$

$$X = A^{-1} P$$

In 2D, let X_i, $i = 1, 2, \ldots, n$, be the pixel values underestimation from attenuation values P_j, $j = 1, 2, \ldots, m$, on the projections. This forms an **over-determined** linear system with $m > n$, wherein A^{-1} is the pseudo-inverse of the matrix A.

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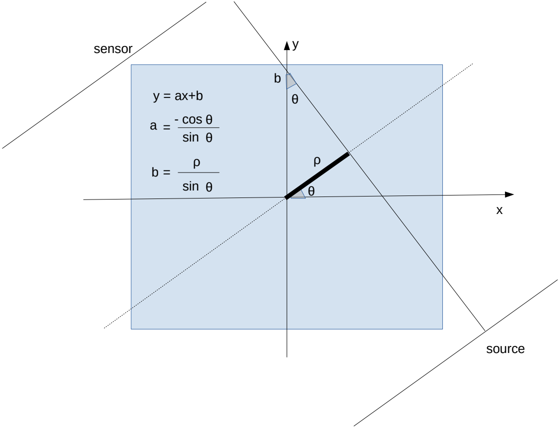
In 2D, for each rotation angle $\theta \in [0, 180)$ of a source-sensor system around the center of the image, the attenuation values $R(\theta, \rho)$ at the polar coordinates (θ, ρ) result from the integration of the pixel values $f(x', y')$ underestimation along a line segment $y = ax + b$ from the source to the sensor.

$$R(\theta, \rho) \;\; = \;\; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x', y')\delta(y' - y)dx'dy'.$$

The line segment $y = ax + b$ is orthogonal to the axis ρ, which is parallel to the source-sensor system, and the function δ (delta of Dirac) considers only the (x', y') coordinates that satisfy the line segment equation.

By changing ρ within $[-\frac{D}{2}, \frac{D}{2}]$, where D is the diagonal of the image, we obtain one projection (signal) for a fixed $\theta \in [0, 180)$. By mapping the projections at each column θ, we can see an attenuation image $R(\theta, \rho)$ with sinusoidal patterns.



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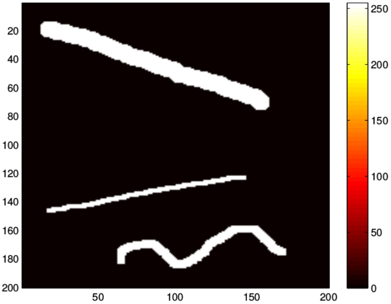

www.cs.cmu.edu/~pmuthuku/mlsp_page/lectures/Carsten_Hoilund_Radon.pdf

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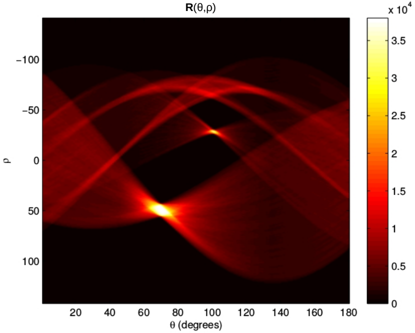

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- This theorem says that the Fourier transform $\mathcal{F}_\theta(w)$ of $R(\theta, \rho)$ for a fixed column θ is equal to the slice (line) $\mathcal{S}(w)$ of the Fourier transform $\mathcal{F}(u, v)$ of $f(x, y)$ passing through its origin and parallel to the projection line.

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In 3D, the interior of the material is **sampled** at integer coordinates (x,y,z), spaced by short distances (d_x,d_y,d_z), and the estimated values $f(x,y,z)$ from 2D projections are **quantitized** with b bits, resulting the image values $I(x,y,z) \in [0, 2^b - 1]$.

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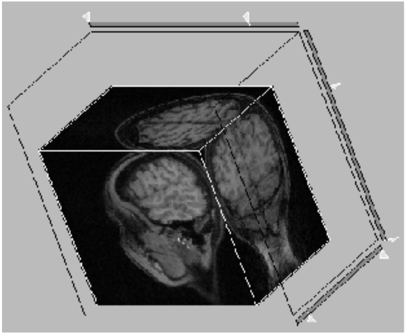

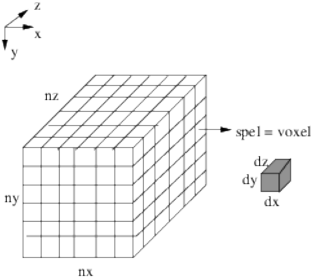

The values $I(x, y, z)$ are also referred to as $I(p)$, where $p = (x_p, y_p, z_p)$ is a *voxel* — space element (spel) in 3D or volumetric space defined by (d_x, d_y, d_z) around (x_p, y_p, z_p).
See **the medical image coordinate systems** in
`www.slicer.org/slicerWiki/index.php/Coordinate_systems`

Image Resolution

- For a same spatial region, lower is the volume $d_x d_y d_z$ of a voxel (e.g., $1mm^3$), higher is the number $n_x n_y n_z$ of voxels, and so higher is the **spatial resolution** of the image.

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- For a same range of measure, shorter is the quantitization interval, higher is the number b of bits, and so higher is the **radiometric resolution**.
- Although CT images are often acquired with values $I(p)$ in $[-1024, 3071]$ (the Hounsfield scale), 3D images are usually acquired with $b = 12$ bits and then they can usually be stored with values in $[0, 4095]$.

3D Image File Formats

- One may find several popular 3D image file formats (e.g., MINC, NIFTI, and DICOM).
- DICOM is the standard one generated by the imaging modality devices, with packages, such as **gdcm**, containing the functions for data manipulation.
  See `http://gdcm.sourceforge.net/wiki/index.php/Main_Page`.

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- The images of the slices can also be compressed, but gdcm provides the decompressing functions.

For the sake of simplicity, we will adopt our file format, named **SCN**, which contains an ASCII header followed by the binary data.

SCN
n_x n_y n_z
d_x d_y d_z
b
binary data...

where $b \in \{8, 16\}$ for one or two bytes per voxel, the voxel size is (d_x, d_y, d_z), and the image size is (n_x, n_y, n_z). The voxels are stored by following the raster order $x = 0, 1, \ldots, n_x - 1$ first, $y = 0, 1, \ldots, n_y - 1$ second, and $z = 0, 1, \ldots, n_z - 1$ third.

3D Image Definition and Representation

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- The values $I(p)$ are stored in a vector, in our case, such that the relation between the vector index $p \in [0, n_x n_y n_z - 1]$ and the voxel coordinates (x_p, y_p, z_p) is given by:

$$
\begin{aligned}
p &= x_p + y_p n_x + z_p n_x n_y \\
z_p &= p \div n_x n_y \\
y_p &= (p \mod n_x n_y) \div n_x \\
x_p &= (p \mod n_x n_y) \mod n_x
\end{aligned}
$$

where $\div$ and $\mod$ are the integer division and the rest of it, respectively.

3D Image Definition and Representation

See

`www.ic.unicamp.br/~afalcao/mo815-3dvis/libmo815-3dvis.tar.bz2`