Volumetric Image Visualization

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Normal estimation for the Phong's illumination model

- In the Phong's illumination model, the angle θ must be computed between $-\mathsf{n}' = -\phi_r^{-1}(\mathsf{n})$ and the normal vector $o.\mathsf{n}(p')$ at the surface point p'.

$$r(p') = k_a r_a + r_d(p')\left(k_d \cos(\theta) + k_s \cos^{n_s}(2\theta)\right).$$

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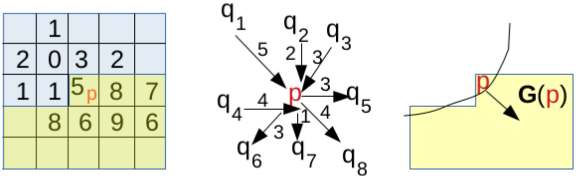
- In this lecture, we will learn when and how to estimate $o.\mathrm{n}(p')$ using the two main approaches:
  - scene-based normal estimation and
  - object-based normal estimation.

Whenever there exists high contrast in $\hat{I} = (D_I, I)$ between an object and its surroundings, the normal vector at a surface voxel $p \in D_I$ can be estimated from the gradient vector G of the scene.

$$
\begin{aligned}
G(p) &= \sum_{\forall q \in \mathcal{A}_r(p)} I(q) - I(p) \frac{q-p}{\|q-p\|}, \\
\mathcal{A}_r(p) &= \{q \in D_I \mid \|q-p\| \leq r, q \neq p\}
\end{aligned}
$$

for small values $1 \leq r \leq 5$.



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- The direction of $\mathrm{G}(p)$ is orthogonal to the surface at p, but its orientation might be towards either the interior or the exterior of the object.
- However, the orientation of the normal vector $o.\mathrm{n}(p)$ should always be towards the exterior of the object.
- Given that we have segmented the objects in the scene and output a label image $\hat{L} = (D_I, L)$, such that $L(p) \in \{0, 1, \ldots, c\}$ indicates when $p \in D_I$ belongs to the background, $L(p) = 0$, or to one of c objects, $L(p) = j$, $j \in [1, c]$, this information can be used as follows.

Scene-based normal estimation

The normal vector $o.\mathrm{n}(p)$ can be defined by

$$o.\mathrm{n}(p) = \begin{cases} \frac{-\mathrm{G}(p)}{\|\mathrm{G}(p)\|} & \text{if } L(p + \alpha\frac{\mathrm{G}(p)}{\|\mathrm{G}(p)\|}) = L(p), \\ \frac{+\mathrm{G}(p)}{\|\mathrm{G}(p)\|} & \text{if } L(p + \alpha\frac{\mathrm{G}(p)}{\|\mathrm{G}(p)\|}) \neq L(p), \end{cases}$$

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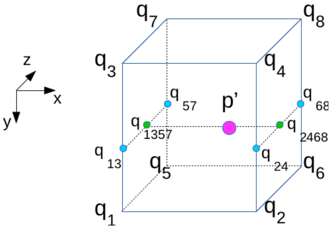
For a point $p' = (x_{p'}, y_{p'}, z_{p'})$ with real coordinates, such that $(\lceil x_{p'} \rceil, \lceil y_{p'} \rceil, \lceil z_{p'} \rceil) \in D_I$, $o.\mathrm{n}(p')$ can be obtained by interpolation from the normal vectors of the nearby spels.

Interpolation for normal estimation

The gradient vectors of the nearby spels $q_k \in D_I$, $k = 1, 2, \ldots, 8$, are used to estimate $G(p')$ and so $o.n(p')$.



$$
\begin{aligned}
G(p') &= (x_{p'} - x_{q_{1357}})G(q_{2468}) + (x_{q_{2468}} - x_{p'})G(q_{1357}) \\
G(q_{2468}) &= (z_{p'} - z_{q_{24}})G(q_{68}) + (z_{q_{68}} - z_{p'})G(q_{24}) \\
G(q_{1357}) &= (z_{p'} - z_{q_{13}})G(q_{57}) + (z_{q_{57}} - z_{p'})G(q_{13})
\end{aligned}
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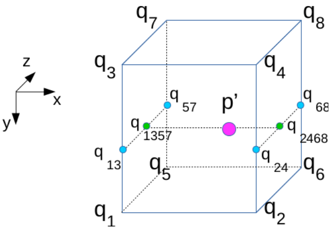
Interpolation for normal estimation



$$
\begin{aligned}
G(q_{24}) &= (y_{p'} - y_{q_4})G(q_2) + (y_{q_2} - y_{p'})G(q_4) \\
G(q_{68}) &= (y_{p'} - y_{q_8})G(q_6) + (y_{q_6} - y_{p'})G(q_8) \\
G(q_{13}) &= (y_{p'} - y_{q_3})G(q_1) + (y_{q_1} - y_{p'})G(q_3) \\
G(q_{57}) &= (y_{p'} - y_{q_7})G(q_5) + (y_{q_5} - y_{p'})G(q_7)
\end{aligned}
$$

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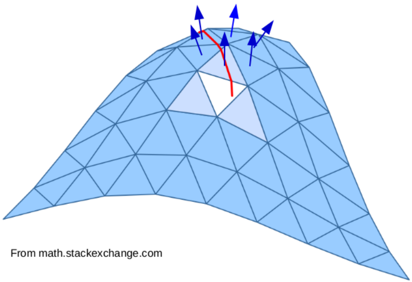
- Bones in MR-T1 images, however, do not present good contrast with the background.
- Such situations require object-based normal estimation methods.
- After segmentation, these methods depend on the type of object representation.
  - Mesh-based object representation.
  - Voxel-based object representation.

Normal estimation from a surface mesh

When the object's surface is represented by a mesh of polygons,

- the vertices are stored in a given order for the normal estimation of the faces from the outer product between two edges.
- Normal estimation for the vertices, along the edges and scan-lines on the faces are obtained by interpolation.



From math.stackexchange.com

Normal estimation from boundary voxels

- When the object's boundary is a voxel set $\mathcal{S} \subset D_I$, the Signed Euclidean distance transform (sEDT) can be used for gradient-based normal estimation.

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- Boundary voxels $\mathcal{S}$ may be extracted from each object o, with $\lambda(o) \in [1, c]$, in $\hat{L} = (D_I, L)$ as

$$\mathcal{S} = \{p \in D_I \ | \ \exists q \in \mathcal{A}_1(p), L(q) \neq L(p), L(p) = \lambda(o)\}$$

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- One sEDT algorithm must be executed for each object o upto a small distance $\rho \geq 1$ from $\mathcal{S}$, such that even the parts of its surface hidden by other objects can be visualized when the visibility of those objects is turned off.

The sEDT algorithm

Input: $\hat{L} = (D_I, L)$, $\lambda(o)$, ρ, and $\mathcal{S}$ of object o.

Output: Signed distance image $\hat{C} = (D_I, C)$, initially with zeros.

For each $p \in D_I$, $C(p) \leftarrow +\infty$.
While $\mathcal{S} \neq \emptyset$ do
    Remove p from $\mathcal{S}$, $C(p) \leftarrow 0$, $R(p) \leftarrow p$, and insert p in Q.
While $Q \neq \emptyset$ do
    Remove $p = arg\min_{q \in Q}\{C(q)\}$ from Q.
    If $\sqrt{C(p)} \leq \rho$ then
        For each $q \in \mathcal{A}_{\sqrt{3}}(p) \mid C(q) > C(p)$ do
            $tmp \leftarrow \|q - R(p)\|^2$.
            If $tmp < C(q)$ then
                If $q \in Q$ then remove q from Q.
                $C(q) \leftarrow tmp$, $R(q) \leftarrow R(p)$, and insert q in Q.
For each $p \in D_I$ do, If $C(p) = +\infty$ then $C(p) \leftarrow 0$. Else,
    If $L(p) \neq \lambda(o)$, then $C(p) \leftarrow -C(p)$.

By construction, the gradient of the signed distance image $\hat{C} = (D_I, C)$ at a boundary voxel $p \in \mathcal{S}$ should always point towards the interior of the object. The normal vector $o.\mathrm{n}(p)$ can then be estimated as

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o.\mathrm{n}(p) &= \frac{-\mathrm{G}(p)}{\|\mathrm{G}(p)\|}, \\
\mathrm{G}(p) &= \sum_{\forall q \in \mathcal{A}_r(p)} C(q) - C(p) \frac{q - p}{\|q - p\|}, \\
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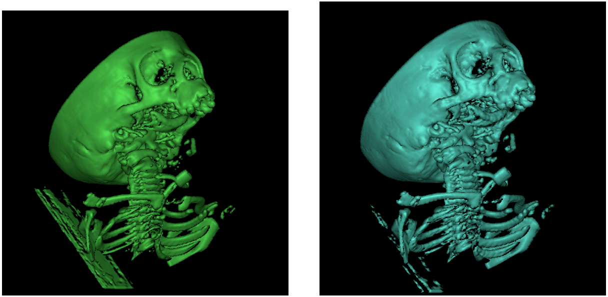
Scene-based vs. Object-based approaches

Scene-based normal estimation provides renditions of better quality, but object-based normal estimation is a good approximation needed in some situations.



Scene-based (left) and object-based (right) normal estimation. The holes may come from segmentation by thresholding or thin parts of the boundary.

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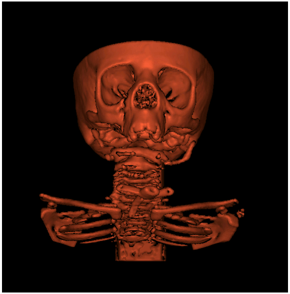

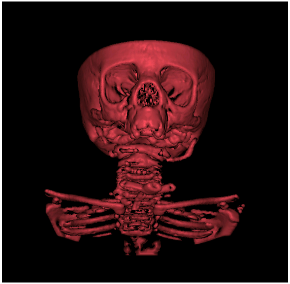

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 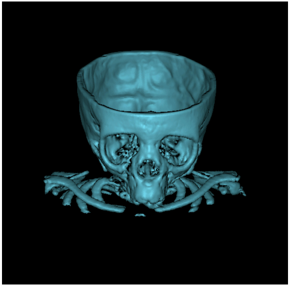

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