Volumetric Image Visualization

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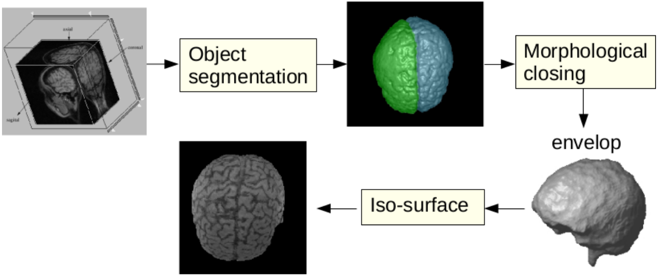
- One can visualize the image texture on iso-surfaces of a 3D object and those renditions are called curvilinear cuts (reformatting) [4].
- This lecture covers the sequence of operations to obtain curvilinear cuts from a 3D object.

Object iso-surfaces and curvilinear cuts

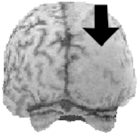
The EDT can be implemented by the IFT algorithm [2] and variants are used for fast morphological operations [1].



The curvilinear cuts are actually obtained from the surface of the object's envelop.

Object iso-surfaces and curvilinear cuts

The visualization can be useful to guide brain resections in the treatment of Epilepsy patients with focal cortical dysplasia [3].



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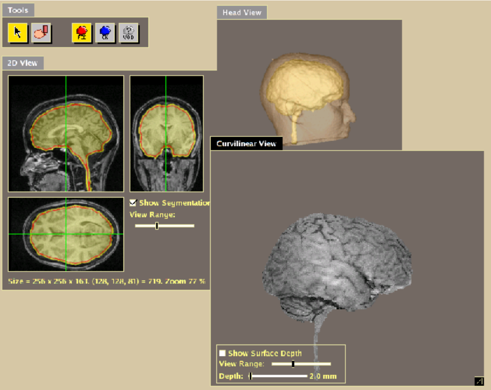

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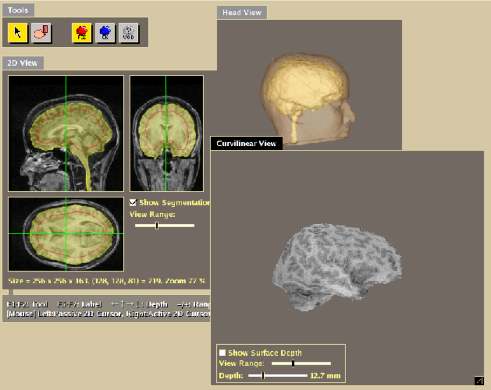

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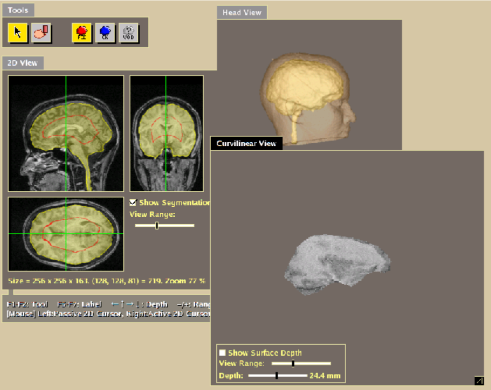

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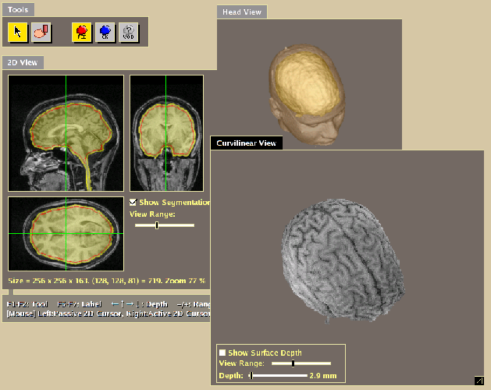

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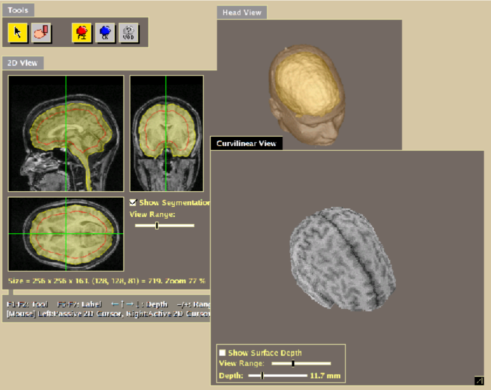

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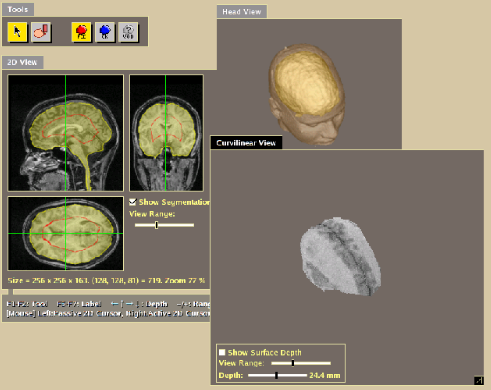

Euclidean distance transform

- Let $\hat{B} = (D_I, B)$ be a binary image of a 3D object (with no holes), such that $B(p) = 1$ inside the object and $B(p) = 0$ outside it.

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\begin{aligned}
\mathcal{S} &: \{p \in D_I \mid B(p) = 1, \exists q \in \mathcal{A}_1(p), B(q) = 0\}, \\
\mathcal{A}_\rho(p) &: \{q \in D_I \mid \|q - p\| \leq \rho\}.
\end{aligned}
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- The IFT algorithm can compute minimum-cost paths from $\mathcal{S}$ such that the cost map C assigns to each voxel $p \in D_I$, the closest distance $C(p)$ between p and $\mathcal{S}$.

Euclidean distance transform

This requires the image graph $(D_I, \mathcal{A}_{\sqrt{3}})$ and path-cost function f,

$$
\begin{aligned}
f(\langle q \rangle) &= \begin{cases} 0 & \text{if } q \in \mathcal{S}, \\ +\infty & \text{otherwise.} \end{cases} \\
f(\pi_p \cdot \langle p, q \rangle) &= \|q - R(p)\|^2,
\end{aligned}
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where $R(p) \in \mathcal{S}$ is the root of the optimum path π_p — i.e., the closest voxel in the object's boundary.

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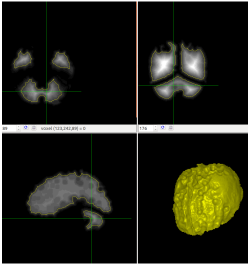

However, morphological closing is needed for the cuts to follow the curvature of the brain.

- The EDT algorithm can be easily modified to propagate either values 1 outside the object (dilation) or values 0 inside it (erosion) up to a given radius $\rho \geq 1$.

Fast morphological operations in binary sets

- The EDT algorithm can be easily modified to propagate either values 1 outside the object (dilation) or values 0 inside it (erosion) up to a given radius $\rho \geq 1$.

- Fast morphological operators can be decomposed into alternate sequences of dilations Ψ_D and erosions Ψ_E.

Fast morphological operations in binary sets

- The EDT algorithm can be easily modified to propagate either values 1 outside the object (dilation) or values 0 inside it (erosion) up to a given radius $\rho \geq 1$.
- Fast morphological operators can be decomposed into alternate sequences of dilations Ψ_D and erosions Ψ_E.
- From the priority queue Q,
  - background seeds for a subsequent erosion can be obtained during dilation and
  - foreground seeds for a subsequent dilation can be obtained during erosion.

Examples of alternate sequences

$$
\begin{aligned}
\Psi_C(\hat{B}, \mathcal{A}_\rho) &= \Psi_E(\Psi_D(\hat{B}, \mathcal{A}_\rho), \mathcal{A}_\rho). \\
\Psi_{CO}(\hat{B}, \mathcal{A}_\rho) &= \Psi_D(\Psi_E(\Psi_E(\Psi_D(\hat{B}, \mathcal{A}_\rho), \mathcal{A}_\rho), \mathcal{A}_\rho), \mathcal{A}_\rho) \\
&= \Psi_D(\Psi_E(\Psi_D(\hat{B}, \mathcal{A}_\rho), \mathcal{A}_{2\rho}), \mathcal{A}_\rho). \\
\Psi_{CO}(\Psi_{CO}(\hat{B}, \mathcal{A}_\rho), \mathcal{A}_{2\rho}) &= \Psi_D(\Psi_E(\Psi_D(\Psi_E(\Psi_D(\hat{B}, \mathcal{A}_\rho), \mathcal{A}_{2\rho}), \\
&\quad \mathcal{A}_{3\rho}), \mathcal{A}_{4\rho}), \mathcal{A}_{2\rho}),
\end{aligned}
$$

where Ψ_C is a closing and Ψ_{CO} is a closing followed by an opening.

For the IFT-based algorithms, let

- $\hat{B} = (D_I, B)$ be the binary image of a presegmented object and $\hat{B}' = (D_I, B')$ may be the resulting dilation/erosion.
- Set $\mathcal{S}$ may represent foreground seeds for dilation or background seeds for erosion.
- C and R are path-cost and root maps.
- Q is a priority queue, $\mathcal{A}_{\sqrt{3}}$ is the adjacency relation for path extension, ρ is a dilation/erosion radius, and *tmp* is a variable.

The EDT algorithm

Input: $\hat{B} = (D_I, B)$ and $\mathcal{S}$ with foreground seeds.
Output: $\hat{C} = (D_I, C)$, initially with zeros.

1. For each $p \in D_I$, if $B(p) = 1$ then $C(p) \leftarrow +\infty$.
2. While $\mathcal{S} \neq \emptyset$ do
3. Remove p from $\mathcal{S}$, $C(p) \leftarrow 0$, $R(p) \leftarrow p$, and insert p in Q.
4. While $Q \neq \emptyset$ do
5. Remove $p = arg\min_{q \in Q}\{C(q)\}$ from Q.
6. For each $q \in \mathcal{A}_{\sqrt{3}}(p) \mid C(q) > C(p)$ and $B(q) = 1$ do
7. $tmp \leftarrow \|q - R(p)\|^2$.
8. If $tmp < C(q)$ then
9. If $q \in Q$ then remove q from Q.
10. $C(q) \leftarrow tmp$, $R(q) \leftarrow R(p)$, and insert q in Q.

Fast dilation

Input: $\hat{B} = (D_I, B)$, $\mathcal{S}$, and ρ.
Output: $\hat{B}' = (D_I, B')$ and seeds $\mathcal{S}$ for erosion.

1. For each $p \in D_I$, $C(p) \leftarrow +\infty$ and $B'(p) \leftarrow B(p)$.
2. While $\mathcal{S} \neq \emptyset$ do
3.   Remove p from $\mathcal{S}$, $C(p) \leftarrow 0$, $R(p) \leftarrow p$, and insert p in Q.
4. While $Q \neq \emptyset$ do
5.   Remove $p = arg\min_{q \in Q}\{C(q)\}$ from Q.
6.   If $C(p) \leq \rho^2$, then $B'(p) \leftarrow 1$.
7.   For each $q \in \mathcal{A}_{\sqrt{3}}(p) \mid C(q) > C(p)$ and $B(q) = 0$ do
8. $tmp \leftarrow \|q - R(p)\|^2$.
9.   If $tmp < C(q)$ then
10.   If $q \in Q$ then remove q from Q.
11. $C(q) \leftarrow tmp$, $R(q) \leftarrow R(p)$, and insert q in Q.
12.   Else, $\mathcal{S} \leftarrow \mathcal{S} \cup \{p\}$.

Fast erosion

Input: $\hat{B} = (D_I, B)$, $\mathcal{S}$, and ρ.
Output: $\hat{B}' = (D_I, B')$ and seeds $\mathcal{S}$ for dilation.

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2. While $\mathcal{S} \neq \emptyset$ do
3.   Remove p from $\mathcal{S}$, $C(p) \leftarrow 0$, $R(p) \leftarrow p$, and insert p in Q.
4. While $Q \neq \emptyset$ do
5.   Remove $p = arg\min_{q \in Q}\{C(q)\}$ from Q.
6.   If $C(p) \leq \rho^2$, then $B'(p) \leftarrow 0$.
7.   For each $q \in \mathcal{A}_{\sqrt{3}}(p) \mid C(q) > C(p)$ and $B(q) = 1$ do
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Implementation issues

- The external boundary $\mathcal{S}$ of an object is defined as

$$\mathcal{S} \quad : \quad \{p \in D_I \mid B(p) = 0, \exists q \in \mathcal{A}_1(p), B(q) = 1\}.$$

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- After 3D object segmentation, an envelop can be created by computing dilation followed by erosion (morphological closing) with radius $\rho = 20$.

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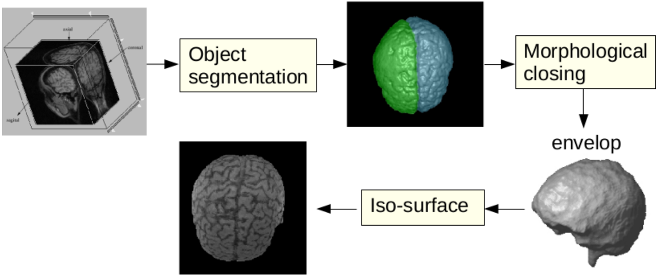
- A zero padding of size ρ is required in order to avoid errors when dilating objects that are closer than ρ to the image's border.

- After 3D object segmentation, an envelop can be created by computing dilation followed by erosion (morphological closing) with radius $\rho = 20$.

- The EDT is then computed for the envelop and curvilinear cuts require the ray casting algorithm up to a desired iso-surface.

Curvilinear cuts

We will learn now how to obtain curvilinear cuts from an input scene $\hat{I} = (D_I, I)$ and the envelop $\hat{E} = (D_I, E)$ of a pre-segmented object in $\hat{I}$.



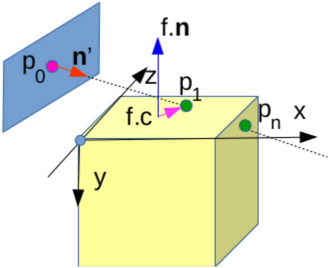
Let ρ be the depth of the cut with respect to the envelop's surface.

Curvilinear cuts

Recall from maximum intensity projection that we must use ϕ^{-1}, being $\phi_r^{-1} = \mathbf{R}_x(-\alpha)\mathbf{R}_y(-\beta)$,

$$\phi^{-1} = \mathbf{T}(x_c, y_c, z_c)\phi_r^{-1}\mathbf{T}(\frac{-d}{2}, \frac{-d}{2}, \frac{-d}{2})$$

on pixels $p = (u_p, v_p, \frac{-d}{2})$ of the viewing plane to map them on points p_0 for ray casting in the direction $\mathbf{n}' = \phi_r^{-1}(\mathbf{n})$.



Recall that, for each ray $p' = p_0 + \lambda \mathbf{n}'$, the intersection points p_1 and p_n with the face planes of the scene must be found by solving the equation

$$\langle p_0 + \lambda \mathbf{n}' - f.c, f.\mathbf{n} \rangle \quad = \quad 0$$

for the six faces $f \in \mathcal{F}$ of the scene, where $f.\mathbf{n}$ and $f.c$ are their

unit normal vector and center point, respectively.

- Now, the DDA algorithm in 3D must be modified to find a point p'_0 at depth ρ in the EDT $\hat{C} = (D_I, C)$ from the surface of the envelop $\hat{E} = (D_I, E)$.

- The algorithm for curvilinear cut is similar to the maximum intensity projection algorithm, except that the DDA function results a point p'_0 whose intensity $I(p'_0)$ must be found by interpolation.

Algorithm to find an iso-surface point p'_0

Input : $\hat{C} = (D_I, C)$, ρ, and $\mathcal{P} = \{p_1, p_n\}$.
Output: Point p'_0 or *nil* for no intersection.

If $p_1 = p_n$ then set $n \leftarrow 1$.
Else
    Set $D_x \leftarrow x_{p_n} - x_{p_1}$, $D_y \leftarrow y_{p_n} - y_{p_1}$, $D_z \leftarrow z_{p_n} - z_{p_1}$.
    If $|D_x| \geq |D_y|$ and $|D_x| \geq |D_z|$ then
        Set $n \leftarrow |D_x| + 1$, $d_x \leftarrow sign(D_x)$, $d_y \leftarrow \frac{d_x D_y}{D_x}$, and $d_z \leftarrow \frac{d_x D_z}{D_x}$.
    Else
        If $|D_y| \geq |D_x|$ and $|D_y| \geq |D_z|$ then

Algorithm to find an iso-surface point p'_0

Set $n \leftarrow |D_y| + 1$, $d_y \leftarrow sign(D_y)$, $d_x \leftarrow \frac{d_y D_x}{D_y}$, and $d_z \leftarrow \frac{d_y D_z}{D_y}$.

Else

Set $n \leftarrow |D_z| + 1$, $d_z \leftarrow sign(D_z)$, $d_x \leftarrow \frac{d_z D_x}{D_z}$, and $d_y \leftarrow \frac{d_z D_y}{D_z}$.

Set $p' = (\lceil x_{p'} \rceil, \lceil y_{p'} \rceil, \lceil z_{p'} \rceil) \leftarrow (x_{p_1}, y_{p_1}, z_{p_1})$.

If $\rho - 0.5 < \sqrt{C(p')} < \rho + 0.5$, then return $p'_0 \leftarrow (x'_p, y'_p, z'_p)$.

For each $k = 2$ to n, do

$p' = (\lceil x_{p'} \rceil, \lceil y_{p'} \rceil, \lceil z_{p'} \rceil) \leftarrow (x_{p'}, y_{p'}, z_{p'}) + (d_x, d_y, d_z)$

If $\rho - 0.5 < \sqrt{C(p')} < \rho + 0.5$, then return $p'_0 \leftarrow (x'_p, y'_p, z'_p)$.

return $p'_0 \leftarrow nil$.

Input : $\hat{I} = (D_I, I)$, $\hat{C} = (D_I, C)$, α, β, and ρ.
Output: Curvilinear cut image $\hat{J} = (D_J, J)$.

$\mathbf{n}' \leftarrow \phi_r^{-1}(\mathbf{n})$, where $\mathbf{n} = (0, 0, 1, 0)$.
For each $p \in D_J$ do
    $p_0 \leftarrow \phi^{-1}(p)$.
    Find $\mathcal{P} = \{p_1, p_n\}$ by solving $\langle p_0 + \lambda \mathbf{n}' - f.c, f.\mathbf{n} \rangle = 0$ for each face $f \in \mathcal{F}$ of the scene, whenever they exist.
    If $\mathcal{P} \neq \emptyset$ then
        $p'_0 \leftarrow \text{FindIsosurfacePoint}(\hat{C}, \rho, \mathcal{P})$
        If $p'_0 \neq nil$ then $J(p) \leftarrow I(p'_0)$ using interpolation.

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